A taste of flavor in string compactifications

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In collaboration with B. Carballo-Pérez & E. Peinado: arXiv:1607.06812

See also more recent progress in arXiv:1708.01595

Our goal

Flavor

Symmetries

The need for discrete flavor symmetries

- Need to explain
  - three flavors of SM particles
  - observed mass patterns
  - CKM, PMNS phases
  - neutrino physics
  - suppression of proton decay
  - ...

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- Spontaneous breakdown of $G \longrightarrow$ phenomenology (predictions?)

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Unfortunately

$\Delta(54)$ is almost unexplored! ☹

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Our purpose HERE:
Explore potential of $\Delta(54)$ and its origin ☺

Flavor symmetries: bottom–up

- **Problem:** Where does G come from? (just artificial?)

Absolutely original tv ad

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  additional discrete symmetries due to **compact dimensions**

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additional discrete symmetries due to **compact dimensions**

geometry $\rightarrow$ symmetry

Discrete non-Abelian flavor symmetries can arise from the geometric structure of string compactifications! ☺

Kobayashi, Nilles, Plöger, Raby, Ratz (2007); Nilles, Ratz, Vaudrevange (2012)

Strings

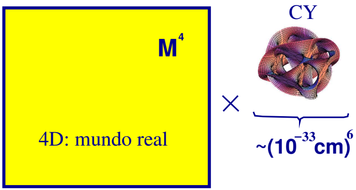
Compactifying string theories

- compactifications:

$$\mathcal{M}^{9,1} = \mathcal{M}^{3,1} \otimes X_6 \qquad \text{vol}(X_6) \sim \ell_{Pl}^6, \quad \mathcal{N} = 1$$

1. X_6: Calabi-Yau threefolds

Candelas, Horowitz, Strominger, Witten (1985)



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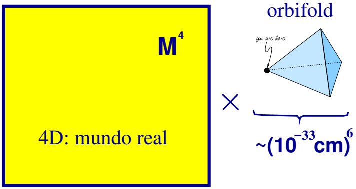
4. X_6: orbifolds — Dixon,Harvey,Vafa,Witten (1985)



Framework: toroidal orbifolds

Orbifold:

$$\mathcal{O} = \frac{\text{compact manifold } \mathcal{M}}{\text{discrete group of isometries } I}$$

Toroidal Orbifold:

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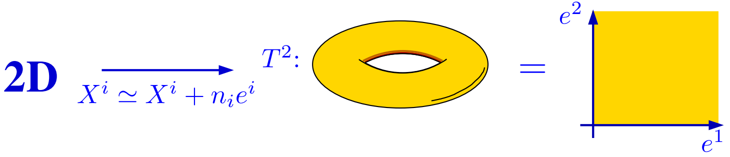

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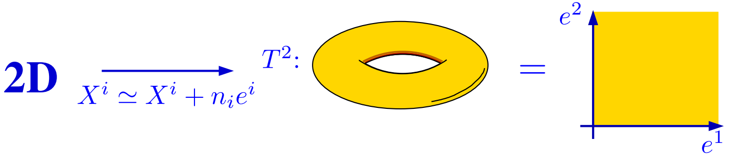
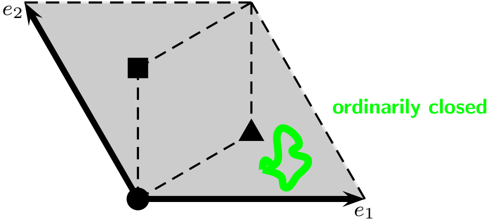


$$\text{then, divide by } I = \begin{cases} \text{Abelian, e.g. } \mathbb{Z}_2, \mathbb{Z}_3, \mathbb{Z}_3 \times \mathbb{Z}_3 \\ \text{non-Abelian} \end{cases}$$

Possible closed strings on the orbifold $T^2/\mathbb{Z}_3$

Three types of *closed strings*:

ordinarily closed, closed on the torus, closed under the orbifold



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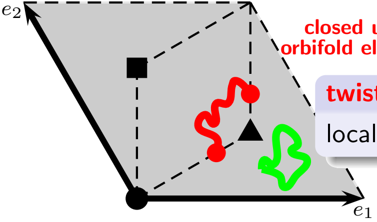

twisted string

localized at fixed point

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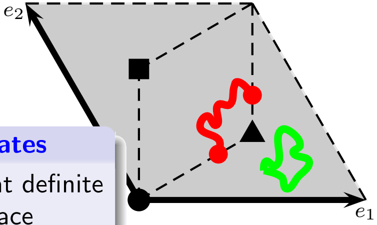

localized twisted states

LEEF states appear at definite points in compact space

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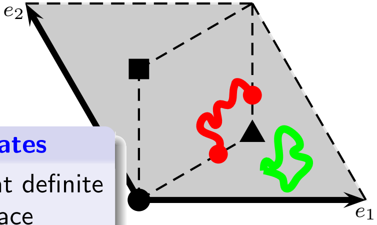

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Twisted states defined by *space group elements*

$g = (\theta, n_\alpha e_\alpha) \longleftrightarrow$ conditions for the twisted strings to close

e.g. $g = (\vartheta = e^{2\pi \mathrm{i}/3}, e_1)$ or $g = (\vartheta^2 = e^{4\pi \mathrm{i}/3}, e_1 + e_2)$

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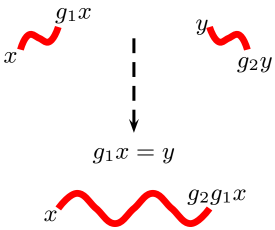

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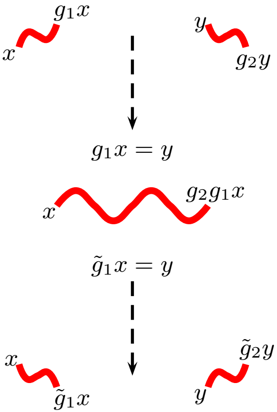

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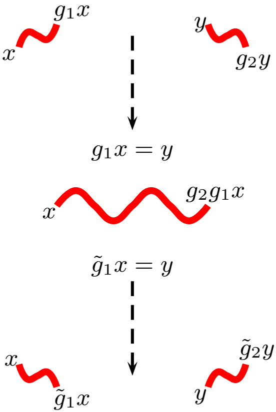

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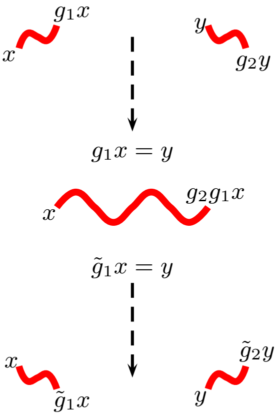

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- See how this works!

An example: flavor symmetry for $T^2/\mathbb{Z}_3$, $\vartheta = e^{2\pi \mathrm{i}/3}$

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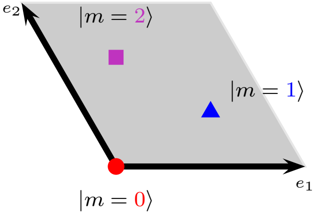

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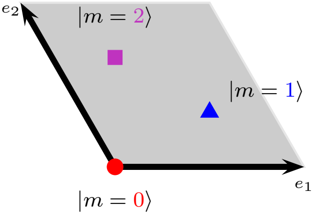

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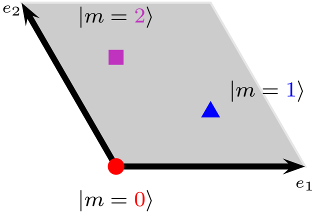

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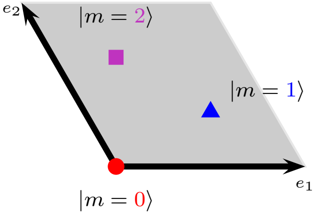

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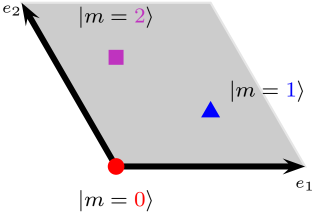

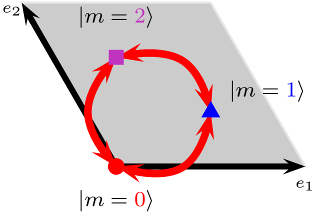

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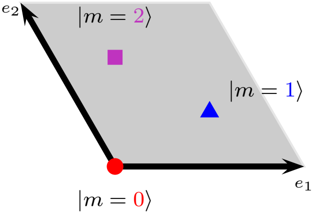
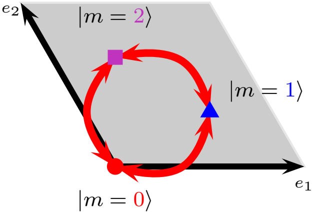


relabeling



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relabeling symmetry:

S_3

Full flavor symmetry group in $T^2/\mathbb{Z}_3$

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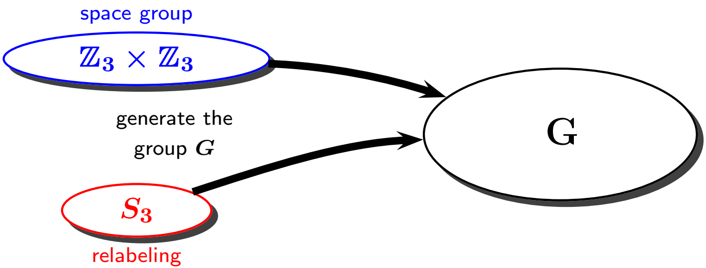

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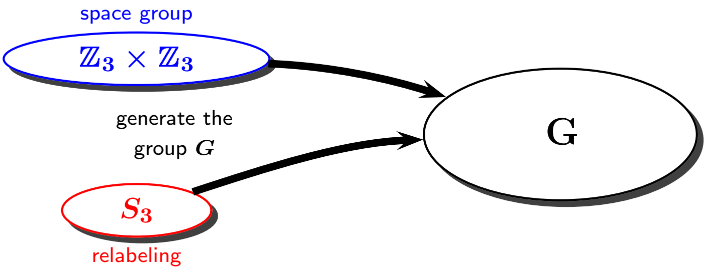

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- There is a *normal subgroup* $N = \mathbb{Z}_3 \times \mathbb{Z}_3 \subset G$

$$G = S_3 \ltimes (\mathbb{Z}_3 \times \mathbb{Z}_3) = \Delta(54)$$

$\Delta(54)$ flavor symmetry in 6D orbifolds

- Construction kit:
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  2. $\left.\begin{array}{lcl}\text{space group rule} & \rightarrow & Z \\ \text{relabeling} & \rightarrow & S\end{array}\right\} \longrightarrow S \cup Z$
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- Simplest options: $T^6/\mathbb{Z}_3$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_2$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$
- BUT no promising models in $T^6/\mathbb{Z}_3$, $T^6/\mathbb{Z}_3 \times \mathbb{Z}_2$ with $\Delta(54)$

Lebedev,Ratz,SR-S,Vaudrevange (2008)

Investigate $\mathbb{Z}_3 \times \mathbb{Z}_3$ orbifolds!

Flavor symmetries in $T^6/\mathbb{Z}_3 \times \mathbb{Z}_3$

- If all fixed points are degenerate, flavor symmetry is large:

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- Models with 2 WLs lead to flavor

$$G = S_3 \ltimes \left[\mathbb{Z}_3^{\vartheta} \times \mathbb{Z}_3^{\omega} \times (\mathbb{Z}_3^{m})^3\right] \supset \boldsymbol{\Delta}(54) \ ☺$$

Procedure to study stringy models with $\Delta(54)$

1. Determine all (inequivalent) $\mathrm{E}_8\times\mathrm{E}_8$ gauge embeddings in heterotic string

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2. Select semi-realistic models
   - 4D gauge group $= \mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times G_{hidden}$
   - 3 generations of quarks and leptons + a pair H_u, H_d
   - $\sin^2\theta_w(M_{GUT}) = 3/8$
   - only SM-vectorlike extra matter

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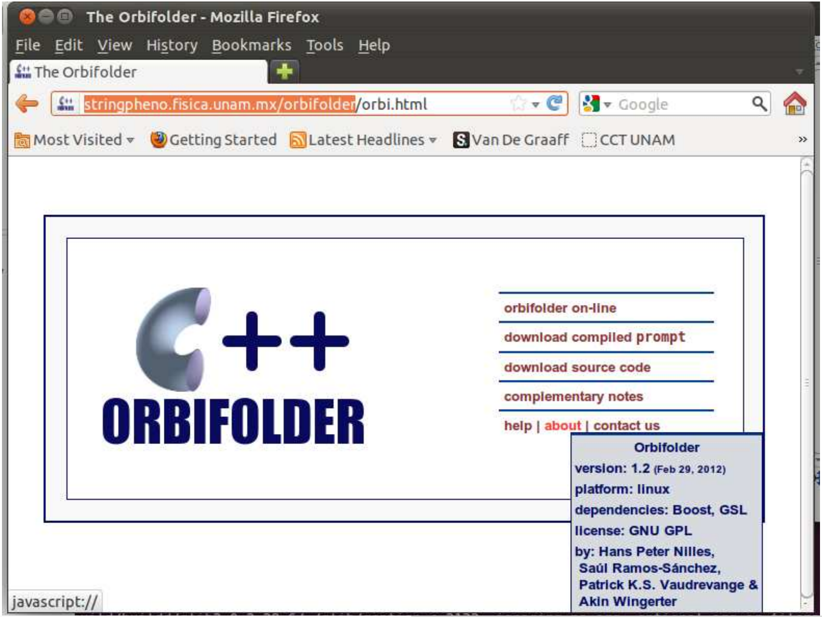
7. Study the low-energy consequences for quarks and leptons

$\rightarrow$ predictions?

Orbifolder needed as a tool (Nilles, SR-S, Vaudrevange, Wingerter, 2011)



Favored flavor symmetries in promising heterotic orbifolds

$\mathbb{Z}_N$		flavor symmetry
$\mathbb{Z}_3$	$(1,1)$	no viable model
$\mathbb{Z}_4$	$(1,1)$	no viable model
	$(2,1)$	$\mathbb{Z}_4$
	$(3,1)$	$\mathbb{Z}_4$
$\mathbb{Z}_6$-I	$(1,1)$	$\mathbb{Z}_6$
	$(2,1)$	$\mathbb{Z}_6$
$\mathbb{Z}_6$-II	$(1,1)$	D_4
	$(2,1)$	D_4
	$(3,1)$	D_4
	$(4,1)$	D_4
$\mathbb{Z}_7$	$(1,1)$	$\mathbb{Z}_7$
$\mathbb{Z}_8$-I	$(1,1)$	$\mathbb{Z}_8$ $[(D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2]$
	$(2,1)$	$\mathbb{Z}_8$ $[(D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2]$
	$(3,1)$	$\mathbb{Z}_8$
$\mathbb{Z}_8$-II	$(1,1)$	D_4 $[(D_4 \times \mathbb{Z}_8)/\mathbb{Z}_2]$
	$(2,1)$	D_4
$\mathbb{Z}_{12}$-I	$(1,1)$	$\mathbb{Z}_{12}$
	$(2,1)$	$\mathbb{Z}_{12}$
$\mathbb{Z}_{12}$-II	$(1,1)$	$\mathbb{Z}_{12}$ $[D_4]$

$\mathbb{Z}_N \times \mathbb{Z}_M$		flavor symmetry
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1,1)$	$(D_4 \times D_4)/\mathbb{Z}_2$ $[D_4]$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1,1)$	$D_4 \times (D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2$
		$[(D_4 \times D_4)/\mathbb{Z}_2]$
		$[D_4$ or $(D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2]$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-I	$(1,1)$	$\mathbb{Z}_2 \times \mathbb{Z}_6$ $[D_4]$
$\mathbb{Z}_2 \times \mathbb{Z}_6$-II	$(1,1)$	no viable model
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1,1)$	$\Delta(54)$
		$[(\Delta(54) \times \Delta(54))/\mathbb{Z}_2]$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1,1)$	$\mathbb{Z}_3 \times \mathbb{Z}_6$ $[\Delta(54)]$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1,1)$	$((D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2)^2/\mathbb{Z}_2$
		$[(D_4 \times \mathbb{Z}_4)/\mathbb{Z}_2]$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1,1)$	$\mathbb{Z}_6 \times \mathbb{Z}_6$

[S.R-S arxiv:1708.01595]

Results of search of models

We found over 7×10^6 admissible gauge embeddings:

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  3. 696 with 2 WLs $\quad \rightarrow \quad \Delta(54) \subset G$ ☺

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explanation for three generations!! ☺

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* Numbers statistically compatible with Nilles, Vaudrevange (2014), but we find many more models ☺

Example

$\Delta(54)$ string spectrum

#	irrep	$\Delta(54)$	label	#	anti-irrep	$\Delta(54)$	label
3	$(\mathbf{3},\mathbf{2})_{\frac{1}{6}}$	$\mathbf{3}_{11}$	Q_i				
3	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}$	$\mathbf{3}_{11}$	$\bar{u}_i$				
3	$(\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{3}_{11}$	$\bar{d}_i$				
3	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{3}_{11}$	L_i				
3	$(\mathbf{1},\mathbf{1})_{1}$	$\mathbf{3}_{11}$	$\bar{e}_i$				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	$\bar{\nu}_i$				
1	$(\mathbf{1},\mathbf{2})_{-\frac{1}{2}}$	$\mathbf{1}_{0}$	H_d	1	$(\mathbf{1},\mathbf{2})_{\frac{1}{2}}$	$\mathbf{1}_{0}$	H_u
			Flavons				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{11}$	ϕ_i^u				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{11}$	$\phi_i^{d,e}$				
3	$(\mathbf{1},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	ϕ_i^{ν}				
2	$(\mathbf{1},\mathbf{1})_{0}$	$2\cdot\mathbf{1}_{0}$	$s^{(d,e)}$, s^u				
128	$(\mathbf{1},\mathbf{1})_{0}$	$77\cdot\mathbf{1}_0+16\cdot\mathbf{3}_{12}+\mathbf{3}_{11}$	N_i				
			Exotic states				
16	$(\mathbf{1},\mathbf{2})_{\frac{1}{6}}$	$10\cdot\mathbf{1}_0+2\cdot\mathbf{3}_{12}$	v_i	16	$(\mathbf{1},\mathbf{2})_{-\frac{1}{6}}$	$4\cdot\mathbf{1}_0+4\cdot\mathbf{3}_{12}$	$\bar{v}_i$
3	$(\mathbf{3},\mathbf{1})_{0}$	$\mathbf{3}_{12}$	y_i	3	$(\overline{\mathbf{3}},\mathbf{1})_{0}$	$3\times\mathbf{1}_0$	$\bar{y}_i$
1	$(\overline{\mathbf{3}},\mathbf{1})_{-\frac{1}{3}}$	$\mathbf{1}_{0}$	z_i	1	$(\mathbf{3},\mathbf{1})_{\frac{1}{3}}$	$\mathbf{1}_{0}$	$\bar{z}_i$
7	$(\mathbf{1},\mathbf{1})_{-\frac{2}{3}}$	$4\cdot\mathbf{1}_0+\mathbf{3}_{12}$	x_i	7	$(\mathbf{1},\mathbf{1})_{\frac{2}{3}}$	$4\cdot\mathbf{1}_0+\mathbf{3}_{11}$	$\bar{x}_i$
51	$(\mathbf{1},\mathbf{1})_{-\frac{1}{3}}$	$30\cdot\mathbf{1}_0+7\cdot\mathbf{3}_{12}$	w_i	51	$(\mathbf{1},\mathbf{1})_{\frac{1}{3}}$	$24\cdot\mathbf{1}_0+9\cdot\mathbf{3}_{12}$	$\bar{w}_i$

$$
\begin{aligned}
W_Y \;&=\; y^u_{ijk} Q_i H_u \bar{u}_j \phi^u_k s_u + y^d_{ijk} Q_i H_d \bar{d}_j \phi^{(d,e)}_k s^{(d,e)} + y^e_{ijk} L_i H_d \bar{e}_j \phi^{(d,e)}_k s^{(d,e)} \\
&+\; y^\nu_{ijkl} L_i H_u \bar{\nu}_j + \lambda_{ijk} \bar{\nu}_i \bar{\nu}_j \bar{\phi}^\nu_k\,, \qquad\qquad i,j,k = 1,2,3,
\end{aligned}
$$

$\Delta(54)$ Phenomenology

$\Delta(54)$ stringy EXAMPLE

- From superpotential, obtain quark and charged-lepton Lagrangian

$$
\begin{aligned}
\mathcal{L}_Y^f &= y_1^f \left[F_1 H \bar{f}_1 \phi_1 + F_2 H \bar{f}_2 \phi_2 + F_3 H \bar{f}_3 \phi_3\right] \\
&+ y_2^f \left[(F_1 H \bar{f}_2 + F_2 H \bar{f}_1)\phi_3 + (F_3 H \bar{f}_1 + F_1 H \bar{f}_3)\phi_2 + (F_2 H \bar{f}_3 + F_3 H \bar{f}_2)\phi_1\right] + h.c.\,,
\end{aligned}
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F_i : LH fermion, f_i : RH fermion, H : Higgs, ϕ_i : flavon scalar

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$$M_f^D \approx \begin{pmatrix} 0 & \sqrt{m_1^f m_2^f} & \frac{m_2^f - m_1^f}{m_3^f}\sqrt{m_1^f m_2^f} \\ \sqrt{m_1^f m_2^f} & m_2^f - m_1^f & 0 \\ \frac{m_2^f - m_1^f}{m_3^f}\sqrt{m_1^f m_2^f} & 0 & m_3^f \end{pmatrix},$$

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For down-quarks

$$\Rightarrow \quad \tan\theta_C \approx \frac{(M_d^D)_{12}}{(M_d^D)_{22}} \approx \sqrt{\frac{m_d}{m_s}}$$

Gatto-Sartori-Tonin ☺

$\Delta(54)$ stringy phenomenology: down sector

Down-quark and charged-lepton sectors are symmetric

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Additional novel consequence

$$\frac{m_s - m_d}{m_b} \overset{!}{=} \frac{m_\mu - m_e}{m_\tau}.$$

more stringent than $b - \tau$ unification ☹

$\Delta(54)$ stringy phenomenology: neutrino sector

From the superpotential W_Y, renormalizable neutrino Lagrangian is

$$\begin{aligned}
\mathcal{L}_Y^\nu &= y_1^\nu \left[L_1 H_u \bar\nu_1 + L_2 H_u \bar\nu_2 + L_3 H_u \bar\nu_3\right] \\
&+ \lambda_1 \left[\bar\nu_1\bar\nu_1\bar\phi_1^\nu + \bar\nu_2\bar\nu_2\bar\phi_2^\nu + \bar\nu_3\bar\nu_3\bar\phi_3^\nu\right] \\
&+ \lambda_2 \left[2\bar\nu_1\bar\nu_2\bar\phi_3^\nu + 2\bar\nu_1\bar\nu_3\bar\phi_2^\nu + 2\bar\nu_2\bar\nu_3\bar\phi_1^\nu\right]
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$\Rightarrow$ type I see-saw possible!! ☺

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$\Rightarrow$ RH neutrino masses masses

$$M_{RH} = \begin{pmatrix} \lambda_1\bar\phi_1^\nu & \lambda_2\bar\phi_3^\nu & \lambda_2\bar\phi_2^\nu \\ \lambda_2\bar\phi_3^\nu & \lambda_1\bar\phi_2^\nu & \lambda_2\bar\phi_1^\nu \\ \lambda_2\bar\phi_2^\nu & \lambda_2\bar\phi_1^\nu & \lambda_1\bar\phi_3^\nu \end{pmatrix}$$

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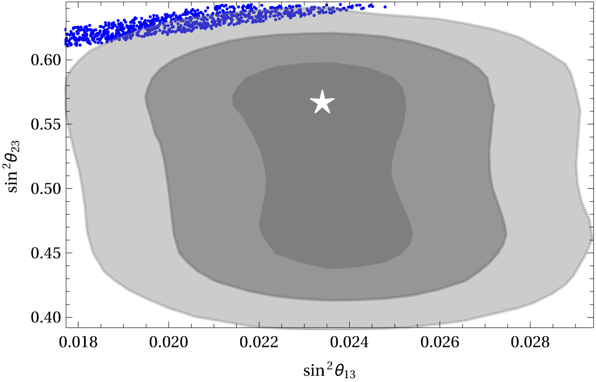
assume a vacuum/flavon alignment: $\langle\bar\phi^\nu\rangle = v_{\nu_3}\,(R_1, \delta, 1)$
run over all parameter values looking for neutrino phenomenology compatible with data

$\Delta(54)$ stringy phenomenology: neutrino sector

Correlation atmospheric–reactor mixing angles compatible with best fit

Forero, Tortola, Valle (2014)



- atmospheric mixing angle: $51.3^o \lesssim \theta_{23} \lesssim 53.1^o$ (second octant) ☺
- reactor mixing angle: $7.8^o \lesssim \theta_{13} \lesssim 8.9^o$ ☺
- $6meV \lesssim m_{\nu_1} \lesssim 6.8meV\,, \quad 65meV \lesssim \sum m_\nu \lesssim 70meV$ ☺

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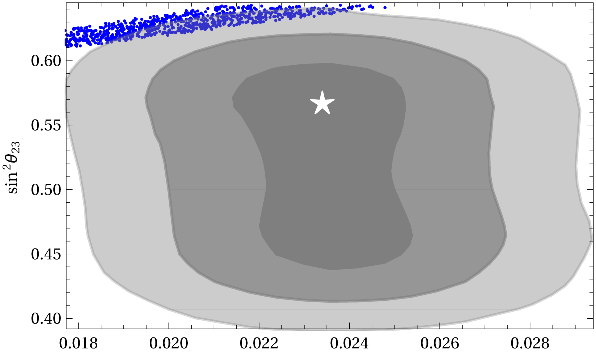


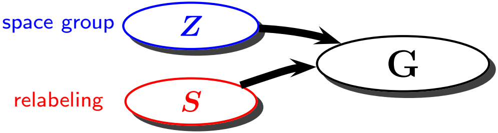
Advantages of model:

Compatible with T2K (2016)
only normal hierarchy admissible
Falsifiable soon

- atmospheric … cond octant) ☺
- reactor mixin
- $6meV \lesssim m_{\nu_1}$ … eV ☺

Conclusions

- $\Delta(54)$ and other flavor symmetries from compactification geometry:



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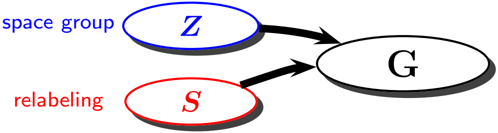

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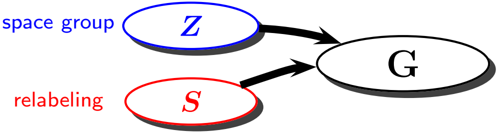

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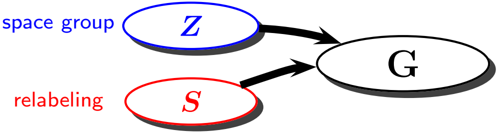

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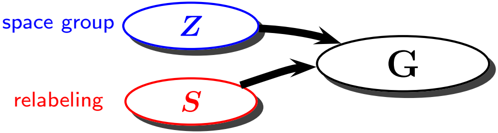

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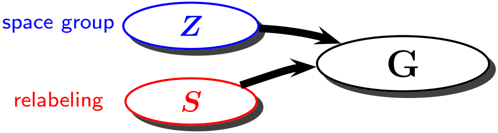

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  4. funny unification mass relation ☹
  5. normal hierarchy for neutrino masses
  6. "predict" neutrino mixing angles ☺

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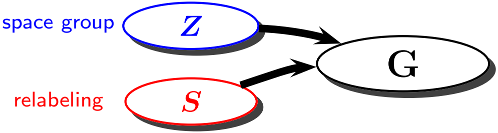

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BUT

- bad quark mixing angles
- proton decay
- unjustified vacuum alignments
- SUSY breaking not understood

And, of course,...