ON THE NON-LINEAR SUPERGRAVITY FROM AN $\overline{D3}$ BRANE

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Introduction

- In 1998, the evidence for a positive cosmological constant was found by the Supernova Cosmological Project and the High-Z Supernova Search Team.
- Such cosmological constant is responsible for the 73 % of the energy of the universe (dark energy).
- Our universe may currently be described by the de Sitter space.
- If String Theory is considered to be a fundamental theory, then it should provide a theoretical framework to describe these results.

Some steps towards this goal:

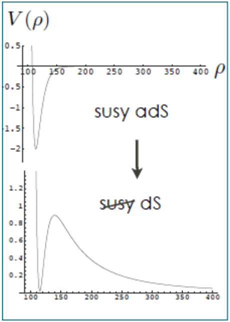
- In GKP, new string vacua can be constructed by flux compactification. It allows to fix the complex structure moduli and dilaton but not the Kähler moduli.

  Giddings, Kachru, & Polchinski '01

- KKLT construct a dS vacua from $\overline{\mathrm{D3}}$-branes in type IIB flux compactification:

  Kachru, Kallosh, Linde & Trivedi '03

The Kähler moduli is fixed by non-perturbative corrections to the potential giving susy adS vacuum.
Uplift to dS by susy breaking $\overline{\mathrm{D3}}$-brane.



The low energy effective theory (LEEFT) $\mathcal{N} = 1$ sugra in 4D is given by

$$K = -3\ln\left(T + \bar{T}\right), \quad W = W_{flux} + Ae^{-aT}$$

and

$$V_F = e^K \left[K^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2 \right]$$

Adding an $\overline{\text{D3}}$-brane (localized at the bottom of a warped throat), gives the up-lifting potential contribution

$$V_{up} = \frac{M^4}{(2\rho)^2}, \qquad \rho = \operatorname{Im} T$$

The total potential escalar

$$V_{tot} = V_F + V_{up} \longrightarrow \text{New ingredient}$$

The $\overline{\text{D3}}$-brane, spontaneously breaks supersymmetry.
Any theory with s.b.s at some scale would have a non-linearly realization of supersupersymmetry as its effective field theory.
Recently, it was understood how the uplifting term can be integrated into an $\mathcal{N} = 1$ non-linear supergravity action.
McGuirk, Shiu & Ye '12, Kallosh & Wrase '14, Bergshoeff, Dasgupta, Kallosh, Van Proeyen & Wrase '15, Kallosh, Quevedo & Uranga '15

Here we would like understand:

- The LEEFT when the $\overline{\text{D3}}$-brane is included from the beginning and it couples to all bulk moduli.
- Explore the modular invariance of the full LEEFT
- Study the robustness of the uplifted 4D dS vacuum against quantum gravity corrections.

Volkov-Akulov action

Volkov & Akulov '72

- Before the discovery of the neutrino oscillations, Volkov and Akulov proposed the massless goldstino as the neutrino.
  The $\mathcal{N} = 1$ VA model consist of a single spinor λ:

$$\mathcal{S}_{VA} = -M^2 \int E^0 \wedge E^1 \wedge E^2 \wedge E^3 \,, \qquad E^\mu = \mathrm{d}x^\mu + \bar{\lambda}\gamma^\mu \mathrm{d}\lambda$$

$$\mathcal{L}_{VA} = -M^2(1 + \bar{\lambda}\gamma^\mu \partial_\mu \lambda) + \dots$$

- It is invariant under the non-linear supersymmetric transformation:

$$\delta_\epsilon \lambda = \epsilon + (\bar{\lambda}\gamma^\mu \epsilon)\partial_\mu \lambda$$

- It involves only a self-interacting fermion with no boson parter.

Volkov-Akulov action

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- Before the discovery of the neutrino oscillations, Volkov and Akulov proposed the massless goldstino as the neutrino.
  The $\mathcal{N} = 1$ VA action for the goldstino:
  $$\mathcal{S}_{VA} = -M^2 \int E^0 \wedge E^1 \wedge E^2 \wedge E^3 \,, \qquad E^\mu = \mathrm{d}x^\mu + \bar{\lambda}\gamma^\mu \mathrm{d}\lambda$$
  $$\mathcal{L}_{VA} = -M^2 (1 + \bar{\lambda}\gamma^\mu \partial_\mu \lambda) + \dots$$
- It is invariant under the non-linear supersymmetric transformation: $\quad \delta_\epsilon \lambda = \epsilon + (\bar{\lambda}\gamma^\mu \epsilon)\partial_\mu \lambda$
- It involves only a self-interacting fermion with no boson parter.
- After the recognition that neutrinos are massive, the VA goldstino played another role in the context of supergravity.

Volkov-Akulov action

The VA model can be described in the superspace formalism of 4d $\mathcal{N} = 1$ supersymemtry in different realizations.

The nilpotent chiral superfield: a chiral superfield S that satisfies $S^2 = 0$

Komargodski & Seiberg '09

$$S = \frac{\psi_S^2}{F_S} + \sqrt{2}\theta\,\psi_S + \theta^2\,F_S$$

The Volkov-Akulov action can be written as:

$$\mathcal{L}_{KS} = \int \mathrm{d}^4\theta S\bar{S} + \int \mathrm{d}^2\theta M^2 S + h.c. \quad \text{with} \quad S^2 = 0$$

After the redefinition $\psi_S = M^2\lambda +$ (non-linear terms)

$$\mathcal{L}_{KS} \equiv \mathcal{L}_{VA}$$

Kuzenko & Tyler '11

Constrained superfield

Rocek '78

$$X^2 = 0 \quad \text{and} \quad -\frac{1}{4} X \bar{D}^2 \bar{X} = M^2 X$$

$$\mathcal{S} = M^2 \int \mathrm{d}^2\theta X + h.c.$$

Constrained vector superfield ($V = V^\dagger$):

Lindström & Rocek '79

Samuel & Wess '83

$$V^2 = 0 \quad \text{and} \quad \frac{1}{16} V D^\alpha \bar{D}^2 D_\alpha V = M^2 V$$

$$\mathcal{S} = M^2 \int \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta} V$$

Different formulations are all equivalent e.g.

$$M^2 V = X \bar{X} \,, \qquad X = -\frac{M}{4} \bar{D}^2 V$$

Can easily be coupled to supergravity
$(\partial_a \to \mathcal{D}_a,\ D_\alpha \to \mathcal{D}_\alpha,\ \bar{D}_{\dot{\alpha}} \to \bar{\mathcal{D}}_{\dot{\alpha}},\ \bar{D}^2 \to \bar{\mathcal{D}}^2 - 8R)$

See e.g. Samuel & Wess '83, Bandos, Heller, Kuzenko, Martucci & Sorokin '16

D3/ $\overline{\text{D3}}$-brane in flat space

The coupling of the 10D bosonic background fields with an D3/$\overline{\text{D3}}$ brane is given by DBI and WZ actions

$$S_{D3/\overline{\text{D3}}} = -T_3 \int d^2\sigma \sqrt{-\det(g_{\mu\nu} + \mathcal{F}_{\mu\nu})} \pm T_3 \int C \wedge e^{\mathcal{F}}$$

which contain the fields: A^μ, X^M and θ_A $\rightarrow$ two 16-component MW spinors

- Such BPS-brane breaks part of the translational symmetry and half of the spacetime supersymmetry (16 supersymmetries).
- These spontaneously broken symmetries are non-linearly realized on the wv.

Hugheee & Polchinski '86,
Callan, Harvey & Strominger '91,

Aganagic, Popescu & Schwarz '96

D3/ $\overline{\text{D3}}$-brane in flat space

The static gauge wv action contains:

- 6 $X^i = \phi^i$ Goldstone bosons
- θ^1 $_{\rightarrow \text{ one 16-component 10D MW spinor}}$ $\rightarrow \lambda^0, \psi^i$
- a vector field A^μ.
- The D3/$\overline{\text{D3}}$ action is invariant under 16 linearly realized transformation associated to $\mathcal{N} = 4$ in 4d.
- The wv fields are included into an $\mathcal{N} = 4$ multiplet.
- But the action is also invariant under 16 non-linear transformations, with λ^0, ψ^i the goldstinos.

The D3/$\overline{\text{D3}}$ is invariant under 16 linear and 16 non-linear supersymmetry transformations

D3/ $\overline{\text{D3}}$-brane in flat space with orientifold.

Wrase & Kallosh '14

Placing the D3-brane on top of an O3, all wv fields are removed and

$$S_{D3} = 0$$

the with the orientifold, supersymmetry is linearly realized on the D3 brane.
For the $\overline{\text{D3}}$-brane, all the bosonic d.o.f are projected out, only one of the 16-component MW spinor θ survive:

$$S_{\overline{\text{D3}}} = -2T_3 \int d^4\sigma \det E \rightarrow \text{VA action}$$

where $E^\mu = \mathrm{d}x^\mu + \sum_{a=0}^{3} \bar{\lambda}^a \gamma^\mu \mathrm{d}\lambda^a$

$\overline{\text{D3}}$ breaks the 4d $\mathcal{N} = 4$ susy spontaneously (non linearly realized) and the 4 λ^a are the 4 goldstinos.

D3/ $\overline{\text{D3}}$-brane in flat space with orientifold.

Wrase & Kallosh '14

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For the $\overline{\text{D3}}$-brane, all the bosonic d.o.f are projected out, only one of the 16-component MW spinor $\lambda \propto \theta$ survive:

$$\mathcal{S}_{\overline{\text{D3}}} = -2T_3 \int d^4\sigma \det E$$

The same results for D3/ $\overline{\text{D3}}$-brane on curved space background

$\overline{\text{D3}}$ in a GKP background

- After compactification

$$SO(9,1) \to SO(3,1) \times SO(6) \to SO(3,1) \times SU(3)$$

- $10D$ $\theta \to 4D$ singlet λ and triplet ψ^i under $SU(3)$.
- To understand the properties of these fields on 4D spacetime filling $\overline{\text{D3}}$ look at:

$$\mathcal{L}_f^{\overline{\text{D3}}} \longrightarrow \text{coupling of fermionic background fields to the brane}$$

- Consider a GKP brackground: F_5,

$$ds_{10}^2 = e^{2A(y)}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-2A(y)}g_{mn}\mathrm{d}y^m \mathrm{d}y^n$$
$$\star G_3 = iG_3 \to \text{ISD}$$

Dimensionally reducing the $\mathcal{L}_f^{\overline{\mathrm{D3}}}$ action in this GKP background to second order in fermions

$$\begin{aligned}\mathcal{L}_{2-f}^{\overline{\mathrm{D3}}} &= 2T_3 e^{4A_0-\phi}\Big[\bar{\lambda}\bar{\sigma}^\mu\nabla_\mu\lambda + \delta_{i\bar{i}}\bar{\psi}^{\bar{i}}\bar{\sigma}^\mu\nabla_\mu\psi^i \\ &\qquad + \lambda\lambda m_0 + m_{ij}\psi^i\psi^j + \psi^i\lambda m_i + h.c.\Big]\end{aligned}$$

where $m_0 \sim G_{0,3}\,,\quad m_{ij} \sim G^P_{2,1}\ ,\ \ m_i \sim G^{np}_{1,2}$

- For supersymmetric background, G_3 should be type (2,1) (vanish (0,3) form)
- Triplet get mass, the singlet λ is massless corresponding to the goldstino of spontaneously broken susy.
- Left with 4d $\mathcal{N}=1$ supersymmetry non-linearly realized.
- What is the corresponding supergravity theory ?

A Simplified KKLT Setup

Bergshoeff, Dasgupta, Kallosh, Van Proeyen & Wrase '15
Vercnocke & Wrase '16
Kallosh, Vernocke & Wrase '16

The story so far:

- Type IIB Calabi-Yau Orientifold compactification with supersymmetric background fluxes and a single $\overline{\text{D3}}$ brane placed on an O3-plane at tip of warped throat.
- $\overline{\text{D3}}$ brane breaks supersymmetry and non-perturbative effects stabilise Kähler modulus:

  $$K = -3\log\left(T+\bar{T}-S\bar{S}\right)\ , \quad W = M^2 S + Ae^{-aT} + Be^{-bT}$$

  where $S^2=0$ carries the massless goldstino, λ, from the wv fermion.
- This gives the uplifted scalar potential:

  $$V = -\frac{3|W(T_0)|^2}{(T_0+\bar{T}_0)^2} + \frac{M^4}{(T_0+\bar{T}_0)^2}$$

  where $D_T W|_{T=T_0} = 0$.
- The triplet of worldvolume fermions can be placed in the contrained chiral superfields Y^i st $SY^i = 0$.

Modular Invariance

Goal: obtain the LEEFT corresponding to a KKLT model, with the same contrained chiral superfields + all light bulk moduli.

Dependence on axion-dilaton and complex structure can be computed using modular invariance inherited from 10D:

- 10D Type IIB classical supergravity action is invariant under the $SL(2,\mathbb{R})$ modular transformations:

$$\tau \to \frac{a\tau - ib}{ic\tau + d}, \quad G_{(3)} \to \frac{G_{(3)}}{ic\tau + d}, \quad F_{(5)} \to F_{(5)}$$

$$\psi_M \to e^{\imath\delta\Gamma_{11}}\psi_M, \quad \text{and} \quad \lambda \to e^{\imath 3\delta\Gamma_{11}}\lambda$$

with $a, b, c, d \in \mathbb{R}$ and $ad - bc = 1$ and

$$e^{\imath\delta} = \frac{(ic\tau + d)^{\frac{1}{4}}}{(-ic\bar{\tau} + d)^{\frac{1}{4}}}$$

.

$\mathcal{N} = 1$ LEEFT of KKLT II

Include the volume fluctuation in the metric:

$$ds_{10}^2 = e^{2A-6u} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{2u-2A} g_{mn} \mathrm{d}y^m \mathrm{d}y^n$$

- In absence of the $\overline{\mathrm{D3}}$, τ and $T = e^{4u} - ib$ fall into chiral multipltes. The LEEFT modular invariant is:

$$K = -\ln(\tau + \bar{\tau}) - 3\ln\left(T + \bar{T}\right) - 3\ln\left(-i\int_M \Omega^\wedge \Omega\right)$$

$$W = \int G_3 \wedge \Omega$$

are invariant under 4D $\tau \to \dfrac{a\tau - ib}{ic\tau + d}$ up to a Kähler transformation:

$$K \to K + \ln|ic\tau + d|^2 \quad \text{and} \quad W \to W/(ic\tau + d)$$

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- D3/$\overline{\mathrm{D3}}$-branes are self-dual: $\Theta \to e^{\imath\delta(\gamma_5 \otimes 1)}\Theta$
  $\Rightarrow \lambda \to e^{-i\gamma_5\delta}\lambda$ and $\psi^i \to e^{-i\gamma_5\delta}\psi^i$

Nilpotent chiral superfield from anti-D3 brane

Write down K, W that matches fermion kinetic and mass terms and scalar potential uplift and satisfies modular invariance:

Nilpotent chiral superfield from anti-D3 brane

Write down K, W that matches fermion kinetic and mass terms and scalar potential uplift and satisfies modular invariance:

$$K = -\ln(\tau+\bar{\tau}) - \ln f(U,\bar{U})$$
$$-3\ln\left(T+\bar{T} - \frac{e^{-4A_0}(T+\bar{T})}{3(\tau+\bar{\tau})f(U,\bar{U})}S\bar{S} - \frac{e^{-4A_0}(T+\bar{T})}{3f(U,\bar{U})^{1/2}}\delta_{ij}Y^iY^j\right)$$

and

$$W = \int G_3 \wedge \Omega + M^2 S + h_{ij}(\tau,U)Y^iY^j + W_{np}$$

with $S^2 = 0$, and $SY^i = 0$ and

$$\psi^S = e^{-\frac{\phi}{2}} e^{-\frac{9u}{2}+\frac{7A_0}{2}} f(U,\bar{U})^{\frac{1}{2}} M^2 \lambda$$

and

$$\psi^i_Y = e^{-\frac{9u}{2}+\frac{7A_0}{2}} f(U,\bar{U})^{\frac{1}{4}} \gamma^0 M^2 \psi^i$$

- Action is modular invariant with

$$S \to \frac{S}{ic + d}$$

and

$$Y^i \to Y^i$$

.

- Uplift in scalar potential is due to an "F-term":

$$V = V_{flux+np} + \frac{|M|^4 e^{4A_0}}{(T + \bar{T})^3}$$

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- Uplift in scalar potential is due to an "F-term":

$$V = V_{flux+np} + \frac{|M|^4 e^{4A_0}}{(T + \bar{T})^3}$$

- Gravitino mass is $m_{3/2} = |e^{K/2} W| = |e^{K/2} W_{np}|$.

Constrained vector superfield from anti-D3 brane

- Action can equivalently be written using:

$$V^2 = 0$$

$$\frac{1}{16}\mathcal{H}(\Phi,\bar{\Phi})V\left(D^{\alpha}(\bar{D}\bar{D}-4R)D_{\alpha}\right)\mathcal{I}(\Phi,\bar{\Phi})V = M^2 V$$

  containing only goldstino – D fixed by $\mathcal{H}(\Phi,\bar{\Phi}),\mathcal{I}(\Phi,\bar{\Phi})$.

- Action has a FI-like coupling – but no gauged shift symmetry!

$$\begin{aligned} K &= -\ln(\tau+\bar{\tau}) - \ln f(U,\bar{U}) \\ &\quad -3\ln\left(T+\bar{T} - \frac{e^{-4A_0}\,(T+\bar{T})}{3f(U,\bar{U})^{\frac{1}{2}}}\delta_{i\bar{i}}Y^i\bar{Y}^{\bar{i}} - \xi V\right) \end{aligned}$$

$$H = T \quad \text{and} \quad W = \int G_3\wedge\Omega + h_{ij}(\tau,U)Y^iY^j + W_{np}$$

where $\lambda_V = e^{-13u/2+3A_0/2}M^2\gamma^0\lambda$ and $\xi = \frac{1}{6}M^2e^{2A_0}$.

- Action is modular invariant with $V \to V$.
- Uplift potential now matched with "D-term" potential, $D = \dfrac{2M^2 e^{4A_0}}{(T+\bar{T})^2}$:

$$\mathcal{L}_D = -\frac{\mathrm{Re}(H)}{2} D^2 - D \frac{\partial K}{\partial V}\bigg|_{V=0} \quad \Rightarrow V_D = \frac{M^4 e^{4A_z}}{(T+\bar{T})^2}$$

- Gravitino mass is $m_{3/2} = |e^{K/2} W| = |e^{K/2} W_{np}|$.

Non-renormalisation theorem

- Textbook non-renormalisation theorem - W cannot depend on Im T or Im τ to all finite orders due to PQ symmetries – then holomorphy implies W cannot depend on Re T or Re τ.

Non-renormalisation theorem

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- Non-renormalisation theorem is implied by m.i.
- $SL(2,\mathbb{R}) \to SL(2,\mathbb{Z})$ after g_s and α' corrections are included, but two relevant subgroups of $SL(2,\mathbb{R})$ are preserved:

  Burgess, Escoda & Quevedo '05

  1. R symmetry: $\psi_\mu \to e^{-i\alpha/2}\psi_\mu$, $\chi \to e^{-3i\alpha/2}$, $\psi_S \to e^{-i\alpha}\psi_S$, $G_3 \to e^{-i\alpha}G_3$ – at leading order in α' – to all orders in g_s. Seiberg '93
  2. PQ symmetry: $\tau \to \tau + ic$ and $T \to T + ic$
- Background fluxes break modular invariance spontaneously - include them as spurion fields.
- Symmetries highly constrain dependence of holomorphic W on τ and T – receives no corrections to all finite

Summary

- 4D LEEFT describing an $\overline{\text{D3}}$-brane in a CY with fluxes and non-perturbative effects is a non-linear sugra, with goldstino falling into a constrained superfield.
- The $\overline{\text{D3}}$-brane "uplift" term can be equivalently parameterised as an F-term uplift or a D-term uplift.
- Modular invariance as a powerful tool to compute dependence of moduli on KKLT effective action including all moduli.
- The non-linear supergravity inherits the stringy modular invariance, which gives rise to non-renormalisation theorems in background fluxes.