The Butterfly Effect in AdS/CFT

Viktor Jahnke

ICN - UNAM

viktor.jahnke@correo.nucleares.unam.mx

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Overview

Introduction

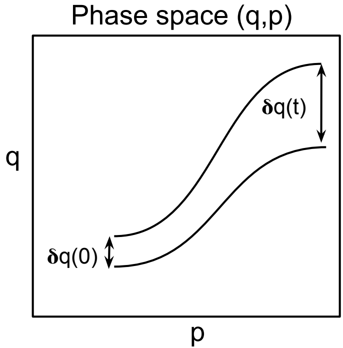
- chaos is a common property of thermal systems;
- in particular, strongly coupled thermal theories described by the boundary theory in AdS/CFT are chaotic;
- chaos-related properties of thermal systems provide us a deeper understanding of AdS/CFT.

Classical Chaos

The distance between nearby trajectories increases exponentially with time

$$|\delta q(t)| \approx e^{\lambda_L t} \, |\delta q(0)|$$

λ_L = Lyapunov exponent



Sensitive dependence on initial conditions

$$\frac{\partial q(t)}{\partial q(0)} = \{q(t), p(0)\}_{\text{Poisson}} \rightarrow \frac{1}{i\hbar}[q(t), p(0))]$$

To avoid phase cancellations, one defines

$$C(t) = -\langle\, [q(t), p(0)]^2 \,\rangle$$

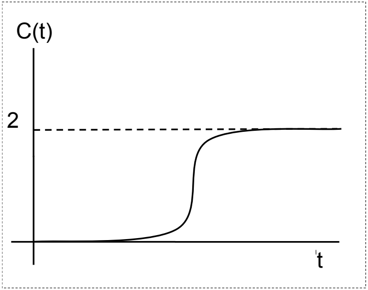
More generally, one replaces

$$q(t) \rightarrow W(t)$$
$$p(0) \rightarrow V(0)$$

where W and V are hermitian operators, and consider

$$C(t) = -\langle\, [W(t), V(0)]^2 \,\rangle_\beta \rightarrow \text{diagnose chaos}$$

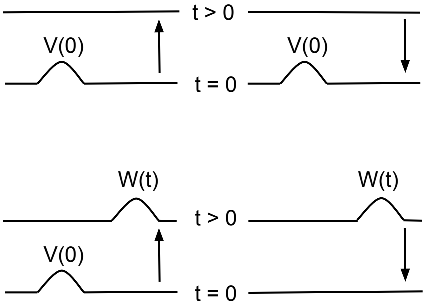
$$C(t) = -\langle\, [W(t), V(0)]^2 \,\rangle_\beta = \langle\, W\, V\, V\, W \,\rangle_\beta + \langle\, V\, W\, W\, V \,\rangle_\beta$$
$$-\langle\, W\, V\, W\, V \,\rangle_\beta - \langle\, V\, W\, V\, W \,\rangle_\beta$$



$\langle\, V\, W\, V\, W \,\rangle_\beta$ or $\langle\, W\, V\, W\, V \,\rangle_\beta \to 0$ at large times.

For a sufficiently chaotic system

$$F(t) = \langle W(t)\, V(0)\, W(t)\, V(0) \rangle_\beta \xrightarrow{t > t_*} 0 \quad \text{diagnose chaos}$$



If, for $t > t_*$, $V(0)$ fails to rematerialize, then $\langle W\ V\ W\ V \rangle \to 0$

t_* = scrambling time

Quantum Chaos

$$F(t) = \langle W_x(t)\, V_0(0)\, W_x(t)\, V_0(0)\rangle_\beta = f_0 - \frac{f_1}{N^2} e^{\lambda_L \left(t - t_* - \frac{x}{v_B}\right)} + \mathcal{O}(N^{-4})$$

- $t_* =$ scrambling time
- $\lambda_L =$ Lyapunov exponent
- $v_B =$ butterfly velocity

These three quantities can be computed in holography by studying shock wave geometries.

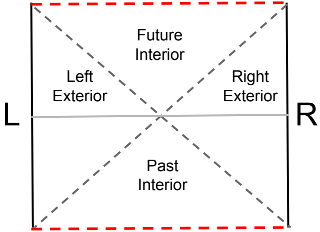
OTOC: $\langle W\, V\, W\, V\rangle \longleftrightarrow$ shock wave collisions in the bulk

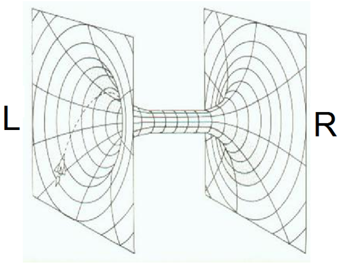
Chaos bound: $\lambda_L \leq \frac{2\pi}{\beta}$. Black holes saturate this bound.

Shenker, Stanford, Maldacena 2015

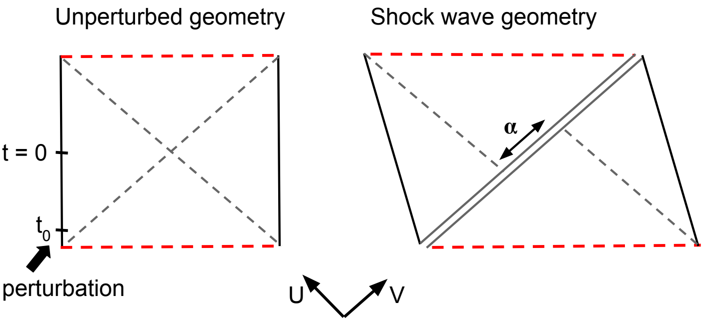
Thermofield double (TFD) = Two-sided black hole (wormhole) Maldacena 2001

$$|TFD\rangle = \frac{1}{Z^{1/2}} \sum_n e^{-\beta E_n/2} |n\rangle_L |n\rangle_R \quad =$$





- highly atypical state;
- there is local entanglement between subsystems of L and R at $t = 0$;
- S_{BH} =total entanglement between L and R.



Metric
$$ds^2 = \frac{\beta^2}{4\pi^2} G_{tt} \frac{dU dV}{U\,V} + G_{ij} dx^i dx^j$$

The metric changes by a shift

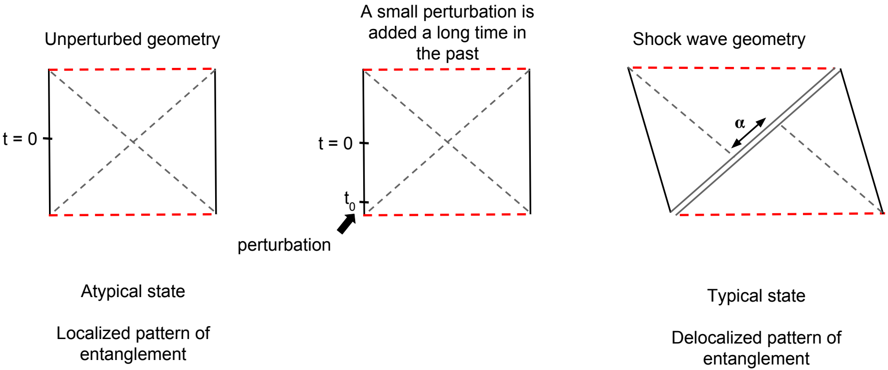
$$V \to V + \theta(U)\,\alpha$$

The key effect is the blue-shift of the infalling perturbation:

$$E(t=0) = \delta E\, e^{\frac{2\pi}{\beta} t_0}, \quad \alpha \sim e^{\frac{2\pi}{\beta} t_0}$$

The Butterfly Effect

Shenker, Stanford 2013



- The two-sided black hole has a local pattern of entanglement at $t = 0$.
- A small perturbation added a long time in the past destroys this local pattern of entanglement.

Mutual Information *as a measurement of local entanglement*

Let A and B be subsystems (regions) of L and R asymptotic boundaries. The mutual information

$$I(A,B) = S(A) + S(B) - S(A \cup B)$$

where $S(A)$ is entanglement entropy of the region A and so on.

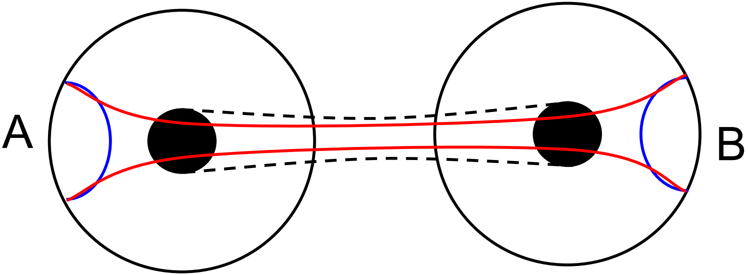
The mutual information provides an upper bound for correlations between A and B

$$\frac{\Big(\langle \mathcal{O}_A \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle\Big)^2}{2 |\langle \mathcal{O}_A \rangle|^2 |\langle \mathcal{O}_B \rangle|^2} \leq I(A,B)$$

Wolf, Verstraete, Hastings 2008

$I(A,B) \rightarrow$ good measure of local entanglement

Mutual Information - two-sided BTZ black hole Shenker, Stanford 2013



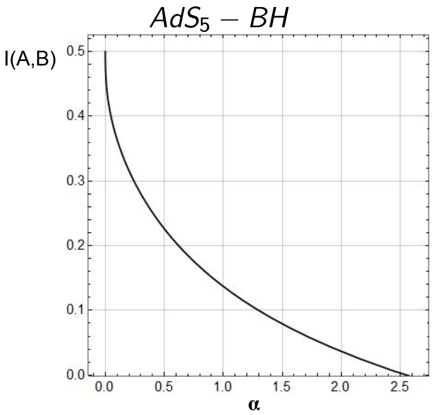
$\gamma_A, \gamma_B =$ blue curves, $\quad \gamma_{A\cup B} =$ red curves $\quad \boxed{I(A,B) = S(A) + S(B) - S(A \cup B)}$

$S(A) = \frac{1}{4G_N}$ Area(γ_A), $\quad S(B) = \frac{1}{4G_N}$ Area(γ_B)

$S(A\cup B) = \frac{1}{4G_N}$ min $\{\text{Area}(\gamma_A) + \text{Area}(\gamma_B), \text{Area}(\gamma_{A\cup B})\}$

$I(A,B) = \frac{1}{4G_N}\left[\text{Area}(\gamma_A) + \text{Area}(\gamma_B) - \text{Area}(\gamma_{A\cup B})\right] \geq 0$ or $I(A,B) = 0$

Disruption of Mutual Information



- BTZ black holes

  Shenker, Stanford 2013

- AdS_{d+1} charged black holes

  Leichenauer 2014

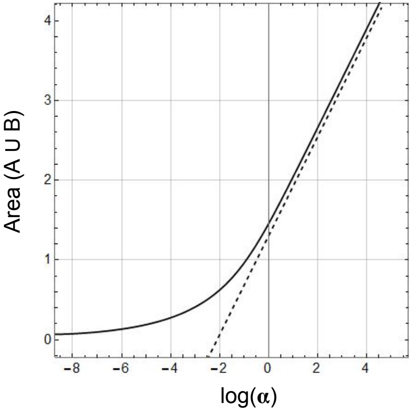
- Dp-branes, Lifshitz, Gauss-Bonnet

  Sircar, Sonnenschein, Tangarife 2016

$I(A,B) \xrightarrow{\alpha\sim 1} 0, \quad \alpha \sim 1$ implies $t_* \approx \frac{\beta}{2\pi}\log S$

$$ds^2 = -G_{tt}dt^2 + G_{uu}du^2 + G_{zz}dz^2 + G_{xx}(dx^2 + dy^2), \quad G_{mn} = G_{mn}(u)$$

$AdS_5 - BH$



Entanglement Velocity v_E

$$\frac{dS_{A\cup B}}{dt} = 2\, V_2\, s_{th}\, v_E$$

where

$$v_E = \frac{\sqrt{-G_{tt}(u_c)\, G_{xx}(u_c)\, G_{ii}(u_c)}}{\sqrt{G_{xx}(u_H)^2\, G_{zz}(u_H)}} \leq v_E^{Schwarzschild}$$

u_c is a point behind the horizon

$$\text{Area } (A\cup B) \approx 4V_2\sqrt{-G_{tt}(u_c)\, G_{xx}(u_c)\, G_{ii}(u_c)}\, \frac{\beta}{4\pi} \log\alpha, \quad \log\alpha = t_0 + \ldots$$

The Butterfly Velocity Roberts, Stanford, Susskind 2014

Localized perturbation: $T_{UU}^{\text{pert}} \approx \delta E\, e^{2\pi t_0/\beta}\, \delta(U)\, \delta(\vec{x})$

The shift $V \to V + \alpha(t, \vec{x})$ is given by

$$(\Box_{\vec{x}} + M^2)\alpha(t, \vec{x}) = \delta E\, e^{2\pi t_0/\beta}\, \delta(\vec{x})$$

For large $|\vec{x}|$, we have

$$\alpha(\vec{x}) \approx \frac{e^{\frac{2\pi}{\beta}(t - t_*) - M|\vec{x}|}}{|\vec{x}|^{\frac{d-1}{2}}}$$

The butterfly velocity is given by

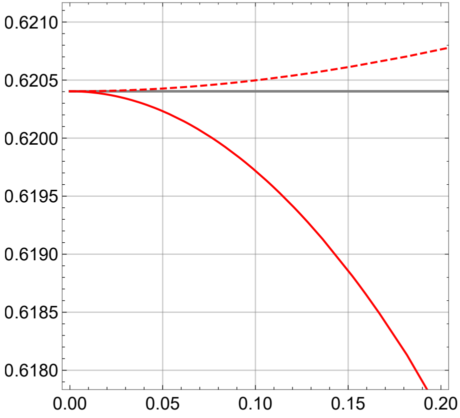
$$v_B = \frac{2\pi}{\beta M} \leq v_B^{\text{Schwarzschild}}$$

$M = M(u_H)$ can be computed in terms of the metric at the horizon.

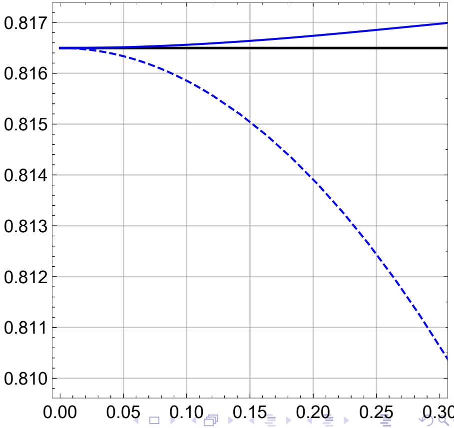
$a =$ anisotropy parameter, $\quad A = B =$ strip-like regions in L and R.

continuous curves = strip orthogonal to the anisotropic direction
dashed curves = strip parallel to the anisotropic direction

$v_E \times a/T$



$v_B \times a/T$

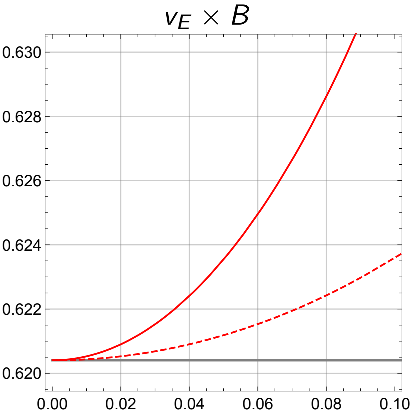


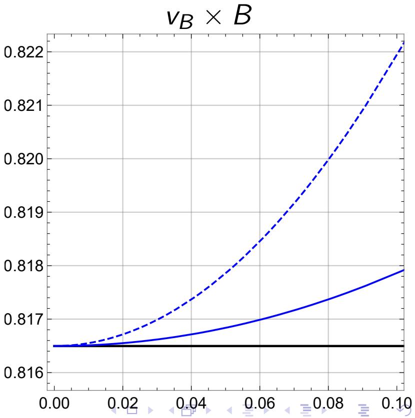
Results for probe D7-branes with $B_{\text{ext}} \neq 0$

Filev, Clifford, Rashkov, Viswanathan 2007

$B \sim$ external magnetic field, $\quad A = B =$ strip-like regions in L and R.

continuous curves = strip orthogonal to the anisotropic direction
dashed curves = strip parallel to the anisotropic direction





Future Directions

- check if the bound for v_E is violated in anisotropic systems in the context of global quenches;
- investigate chaos-related properties of higher curvature gravity theories like
  - Quasi-topological gravity
  - Horndeski gravity
  - Lovelock gravity

THE END