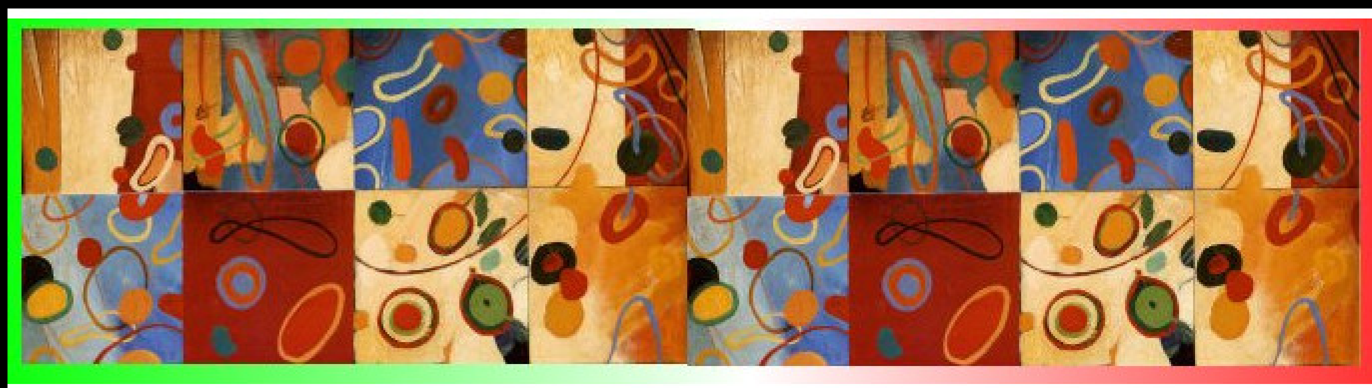
MEXICUERDAS 2017



¡BIENVENIDOS! WELCOME! BEM-VINDOS!

History of Mexicuerdas

- **2008: San Carlos (beach resort in Sonora)**
- **2009: Instituto de Ciencias Nucleares, UNAM**
- **2010: Universidad de Colima**
- **2011: Universidad de Guanajuato**
- **2012: Instituto de Física, UNAM**
- **2013: Universidad de Guanajuato**
- **2014: Universidad de Colima**
- **2015: Universidad de Guanajuato**
- **2016: Facultad de Ciencias, UNAM**
- **2017: Instituto de Ciencias Nucleares, UNAM**
- **2018: CINVESTAV**
- …

We're **celebrating** our
10th Mexicuerdas!!

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- **2017: Instituto de Ciencias Nucleares, UNAM**
- **2018: CINVESTAV**
- …

And next year we'll celebrate

10-year anniversary!!

2009

2016

3 events:

Mexicuerdas (national miniconference)

\+ **Mexican School on Strings and Supersymetry**

\+ **Mextrings** (international miniconference)

have given us a **sense of community**,

and allowed us to **grow stronger**

3 events:
Mexicuerdas (national miniconference)
+ **Mexican School on Strings and Supersymetry**
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have given us a **sense of community**,
and allowed us to **grow stronger**

While we celebrate, it is important to keep in mind that **there is still some distance to go, to fully insert our community in the international community**

This is a **pending task** for us,
and **especially for you, our students**, who
MUST work hard to **be better than we are**!

MEXICUERDAS 2017

	Monday 7	Tuesday 8
9:45-10	*Bienvenida*	
10-11	Alberto Güijosa	Norma Quiroz
11-12	Saul Ramos-Sanchez	Juan Pedraza
12-12:30	Coffee	Coffee
12:30-1:30	Elena Caceres	Edel García
1:30-2:00	Oscar Loaiza	Nana Cabo Bizet
2:00-2:30	David Vergara	Viktor Jahnke
2:30-4:30	Lunch	Lunch
4:30-5:30	Mariano Chernicoff	Daniel Ávila
5:30-6:00	Anderson Misobuchi	Anayeli Ramirez

For the first time we have **4 FEMALE SPEAKERS**!!

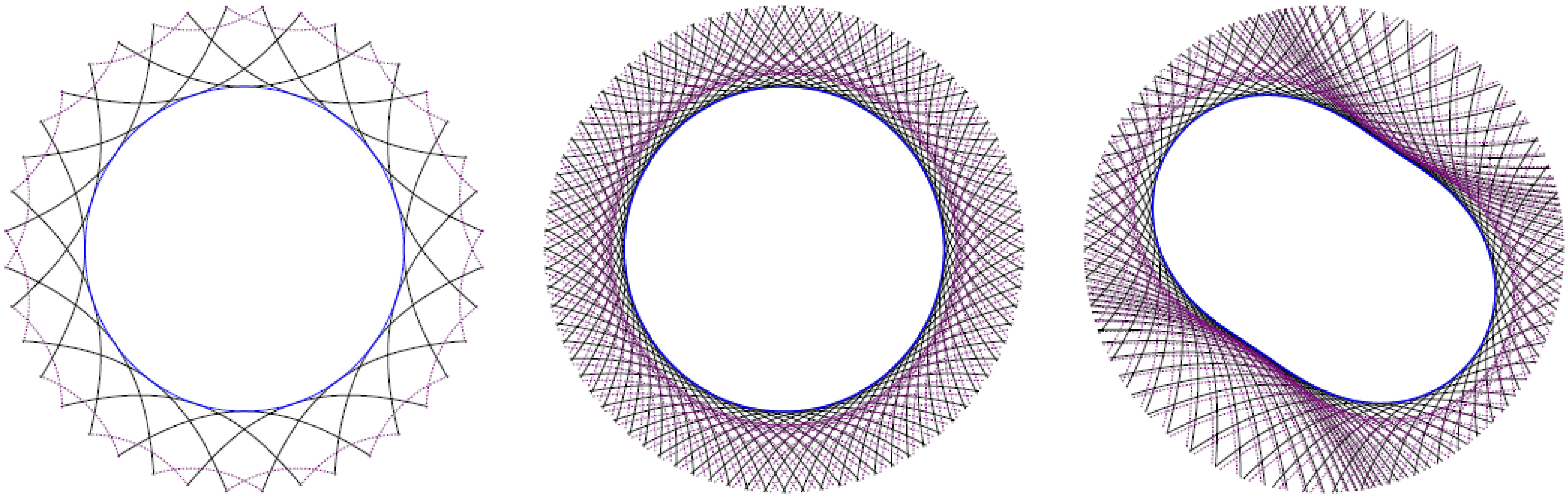
MEXICUERDAS 2017

We have a full program, so we'll have to be rigorous in **sticking to the allotted time for each talk**

Other than that, Mexicuerdas has always been **VERY INFORMAL**, so **please interrupt & ask questions**!!
(If you prefer, **feel free to ask in Spanish**, and we'll translate)

Sorry, but you're **stuck with me** for the first talk!

HOLE-OGRAPHY AND WEDGE RECONSTRUCTION IN ADS



ALBERTO GÜIJOSA HIDALGO
DEPARTAMENTO DE FÍSICA DE ALTAS ENERGÍAS
INSTITUTO DE CIENCIAS NUCLEARES, UNAM
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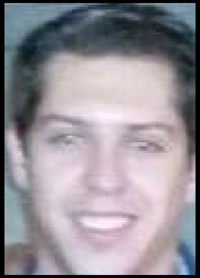
Our story here will be a contribution to the program of **`HOLE-OGRAPHY’**, which aims to **CONSTRUCT SPACETIME USING QUANTUM ENTANGLEMENT**, in the context of **HOLOGRAPHY (= AdS/CFT)**

Based on 2 forthcoming papers:


1) R. Espíndola, AG, A. Landetta, J.F. Pedraza
2) R. Espíndola, AG, -------------- J.F. Pedraza

INTRODUCTION

Quantum ENTANGLEMENT

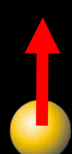
2 or more quantum objects can be ‘undecided’ separately, or **SHARE THEIR INDECISION**

Mathematically, the issue is whether or not their state can be **FACTORIZED**

E.g., for two spin ½ particles:

NOT sharing indecision $$|\psi\rangle = \left(|\uparrow\rangle_L + |\downarrow\rangle_L\right) \otimes \left(|\uparrow\rangle_R + |\downarrow\rangle_R\right)$$ **Unentangled**

Sharing indecisión $$|\psi\rangle = |\uparrow\rangle_L \otimes |\downarrow\rangle_R + |\downarrow\rangle_L |\uparrow\rangle_R$$ **ENTANGLED**



This type of correlation is known as **ENTANGLEMENT**

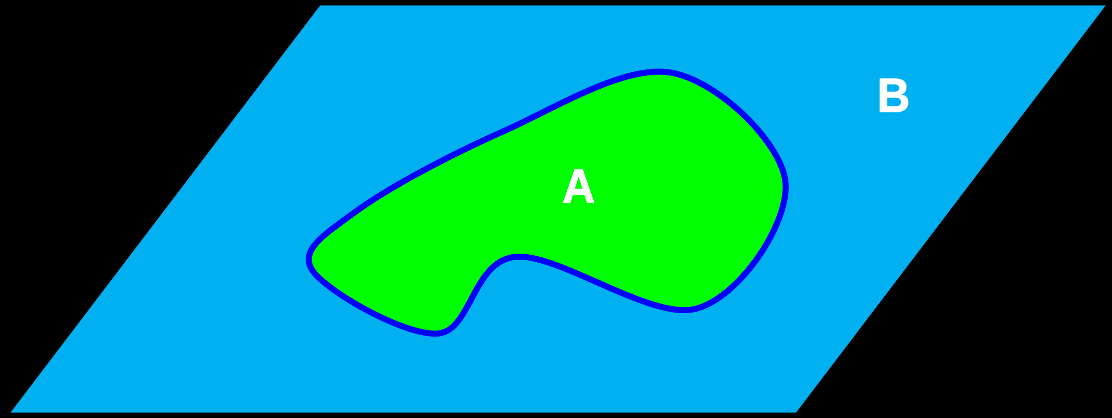
Entanglement Entropy

- Divide the system into **2 SUBSYSTEMS,** A and B
- Given **A STATE**, average over degrees of freedom of B
- Degrees of freedom of A are described then with a **reduced density matrix**

$$\rho_A \equiv \mathrm{Tr}_B(\rho_{\mathrm{tot}}) = \mathrm{Tr}_B(|\psi\rangle\langle\psi|) \equiv \sum_b \langle b|\psi\rangle\langle\psi|b\rangle$$

This describes a **PURE** state iff
A and B were NOT entangled

Basis for B

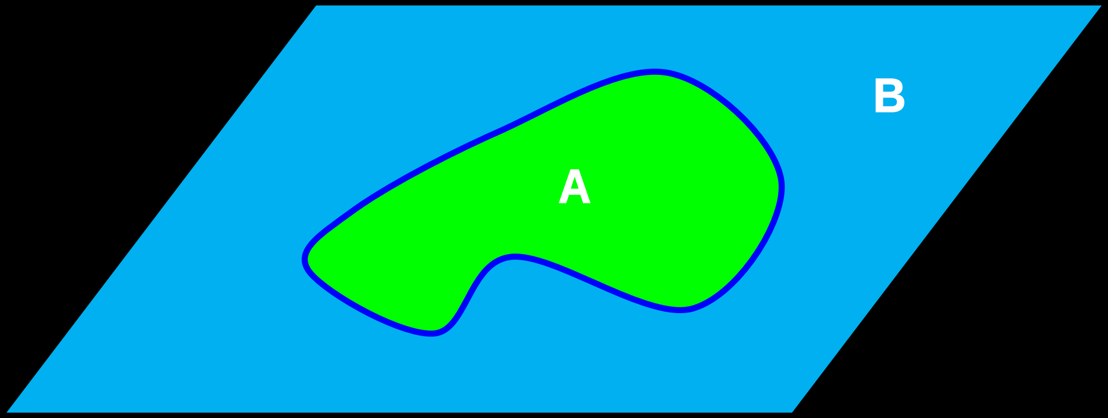


Entanglement Entropy

- Divide the system into **2 SUBSYSTEMS,** A and B
- Given **A STATE**, average over degrees of freedom of B
- Quantify entanglement with **von Neumann entropy**

$$\boxed{S(A) = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)}$$

This measures how NON-pure the state ρ_A is, and therefore, how entangled A and B are IN THE GIVEN STATE



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E.g.,

$$\begin{aligned} |\psi'\rangle &= \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\Big) \\ &= \frac{1}{2}\Big(|\uparrow\rangle + |\downarrow\rangle\Big) \otimes \Big(|\uparrow\rangle + |\downarrow\rangle\Big) \qquad \Longrightarrow \qquad S_{EE} = 0 \end{aligned}$$

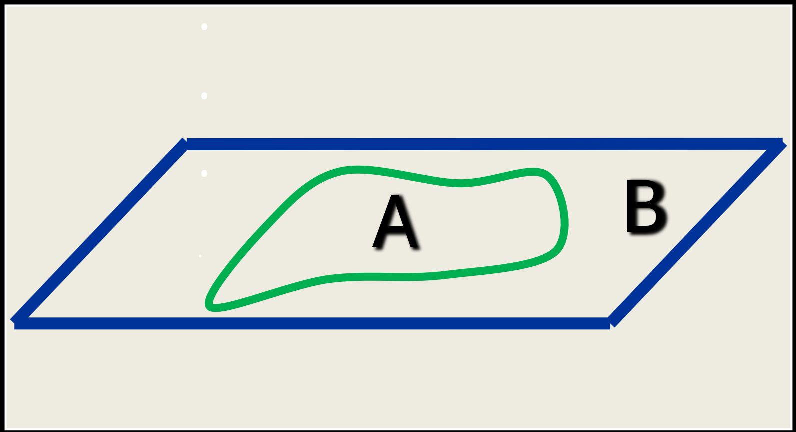
$$|\psi''\rangle = \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle - |\downarrow\downarrow\rangle\Big) \qquad \Longrightarrow \qquad S_{EE} = \log 2$$

Entanglement in QFT

In recent years it has become clear that it is VERY useful to analyze **quantum field theories** (QFTs) using **entanglement** as a tool

[Calabrese,Cardy; Casini,Huerta; Wolf,Verstraete,Hastings,Cirac; Osterloh,Amico,Falci,Fazio; Osborne,Nielsen; Verstraete, Popp,Cirac; Plenio,Eisert,Dreißig,Cramer; Vidal,Latorre,Rico,Kitaev; etc.]

Take a state of our fields, e.g., the **VACUUM** (=state of minimal energy ≠ nothing), and quantify the entanglement between a **SPATIAL** region A and its complement B using **ENTANGLEMENT ENTROPY**



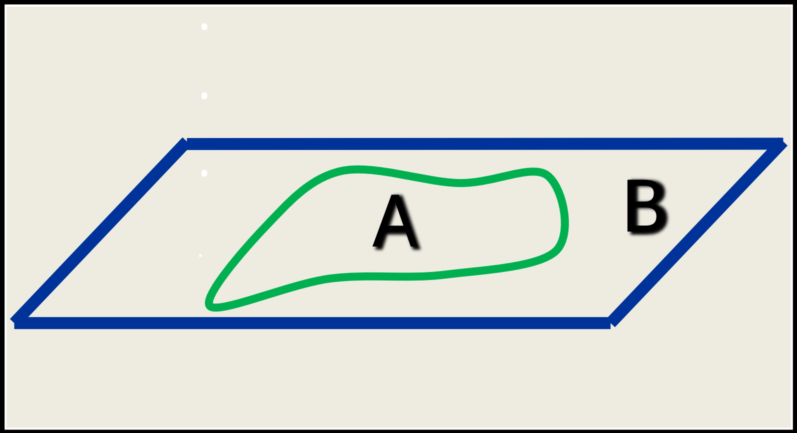
$$S(A) = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement in QFT

Entanglement entropy in QFT is **UV divergent** (due to infinite number of degrees of freedom), and must be REGULARIZED

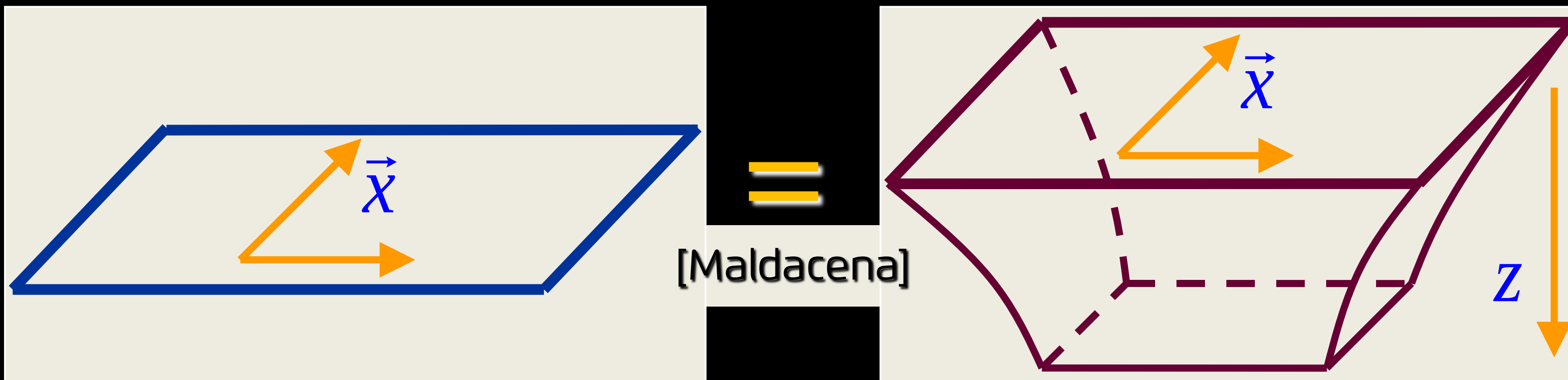
And it is **DIFFICULT TO COMPUTE**, even at weak coupling!

Take a state of our fields, e.g., the **VACUUM** (=state of minimal energy ≠ nothing), and quantify the entanglement between a **SPATIAL** region A and its complement B using **ENTANGLEMENT ENTROPY**



$$S(A) = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

HOLOGRAPHIC CORRESPONDENCE



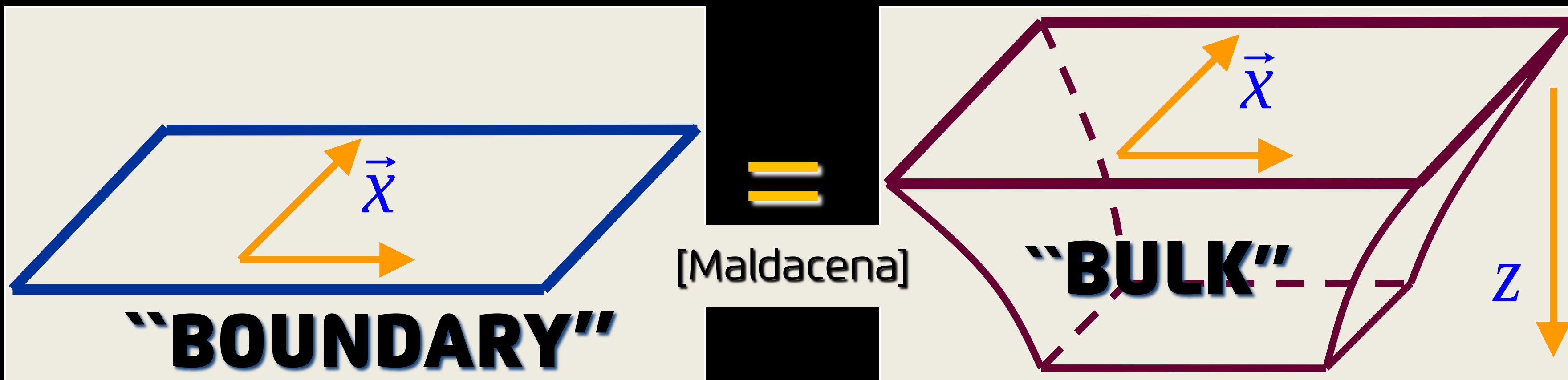
QUANTUM FIELD THEORY
in **d spacetime dim**
with **rigid** metric

QUANTUM GRAVITY THEORY
in (at least) **d+1 dim**
(with **dynamical** metric)

ENERGY scale
(resolution scale)

Extra **dimension of SPACE**
(`radial’ direction)

HOLOGRAPHIC CORRESPONDENCE



QUANTUM FIELD THEORY
in **d spacetime dim**
with **rigid** metric

QUANTUM GRAVITY THEORY
in (at least) **d+1 dim**
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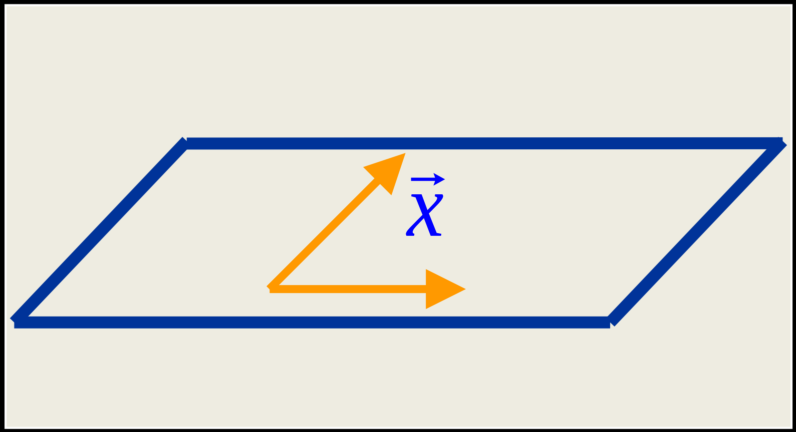
CONFORMAL FIELD THEORY
(CFT = scale-independent)

QUANTUM GRAVITY THEORY
on anti-de Sitter (AdS)

VACUUM of CFT

Pure AdS geometry

HOLOGRAPHIC CORRESPONDENCE



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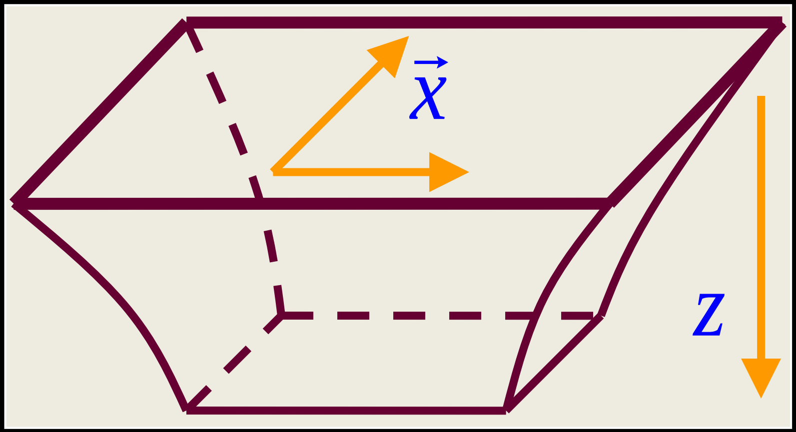


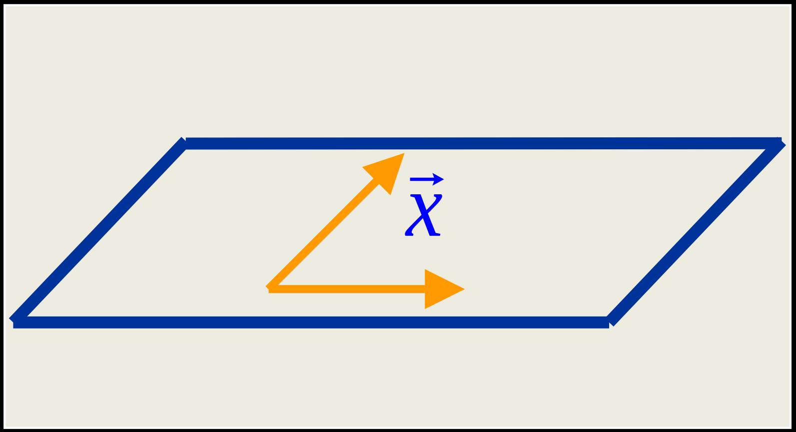
QUANTUM FIELD THEORY
in **d spacetime dim**
with **rigid** metric

QUANTUM GRAVITY THEORY
in (at least) **d+1 dim**
(with **dynamical** metric)

Hard calculations:
LARGE number of
degrees of fredom
STRONG coupling

Easy calculations:
CLASSICAL gravity
WEAKLY CURVED geometry

HOLOGRAPHIC CORRESPONDENCE



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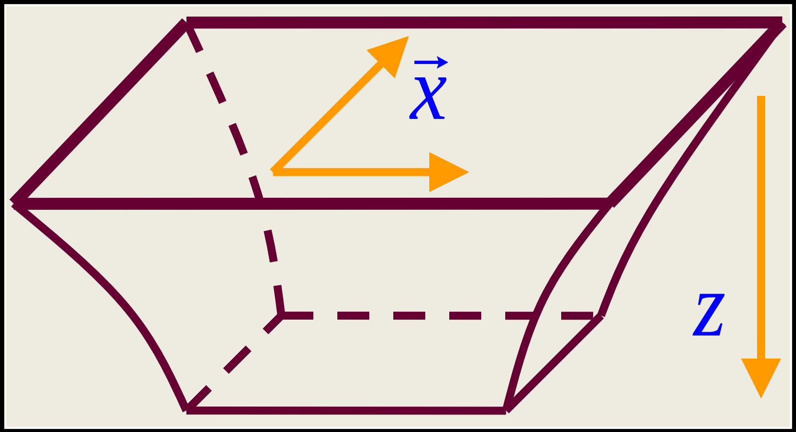


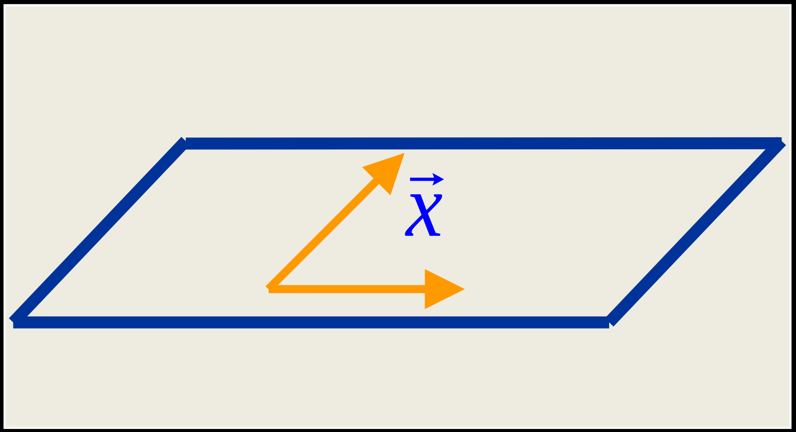
QUANTUM FIELD THEORY
in **d spacetime dim**
with **rigid** metric

QUANTUM GRAVITY THEORY
in (at least) **d+1 dim**
(with **dynamical** metric)

←

In this direction, gravity theory is used as **a tool** to understand some things that previously we DID NOT understand

HOLOGRAPHIC CORRESPONDENCE



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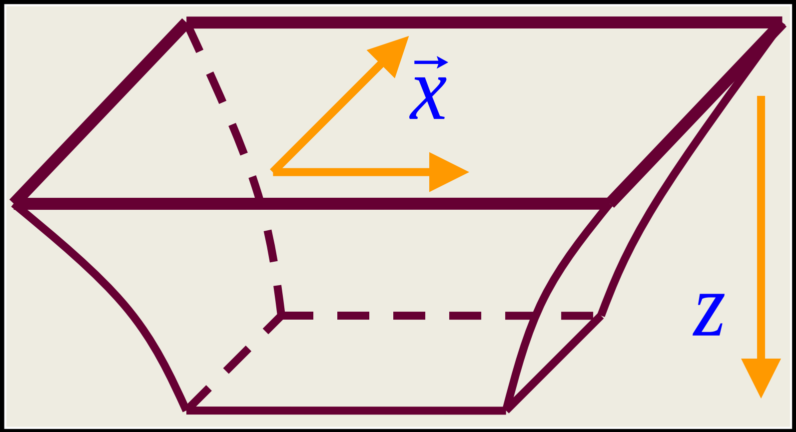


QUANTUM FIELD THEORY
in **d spacetime dim**
with **rigid** metric

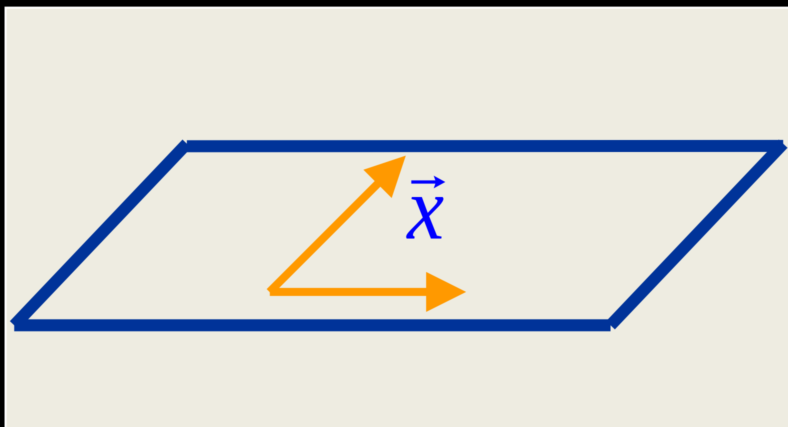
QUANTUM GRAVITY THEORY
in (at least) **d+1 dim**
(with **dynamical** metric)

$\longrightarrow$

In the opposite direction, a **MIRACLE** is taking place:

HOW CAN GRAVITY EMERGE FROM QFT??!!

HOLOGRAPHIC CORRESPONDENCE



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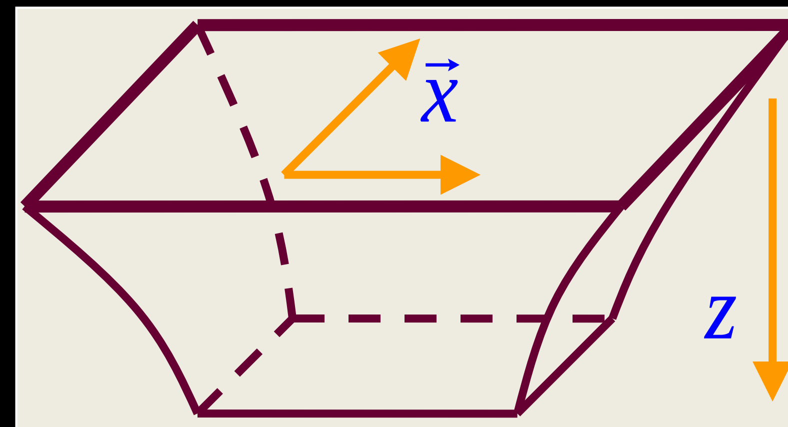

QUANTUM FIELD THEORY
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QUANTUM GRAVITY THEORY
in (at least) **d+1 dim**
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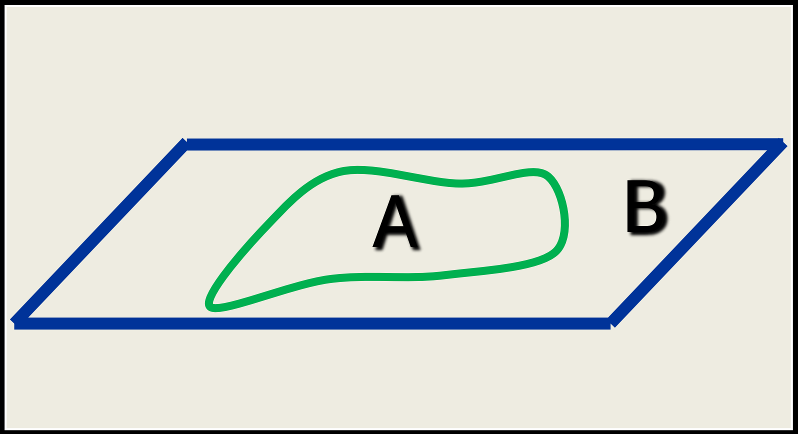
→

In the opposite direction, a **MIRACLE** is taking place

This is the question of `Bulk RECONSTRUCTION'

Entanglement in Holography



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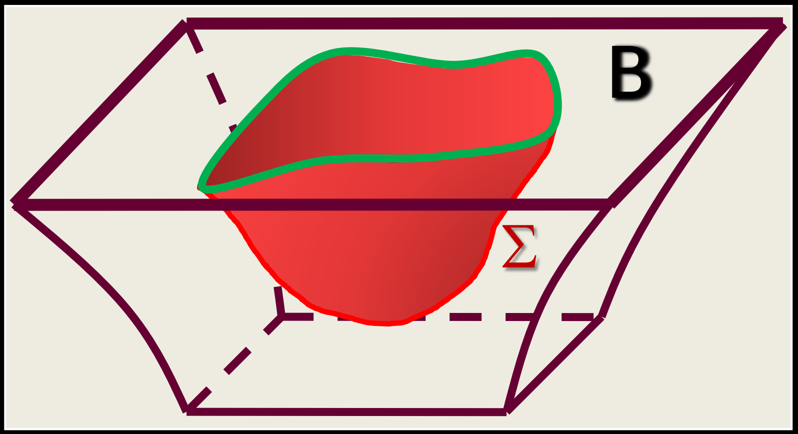
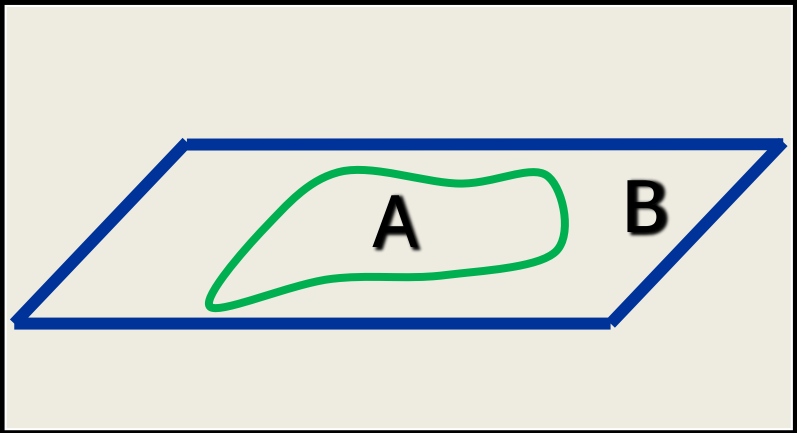


$$S(A) = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement in Holography



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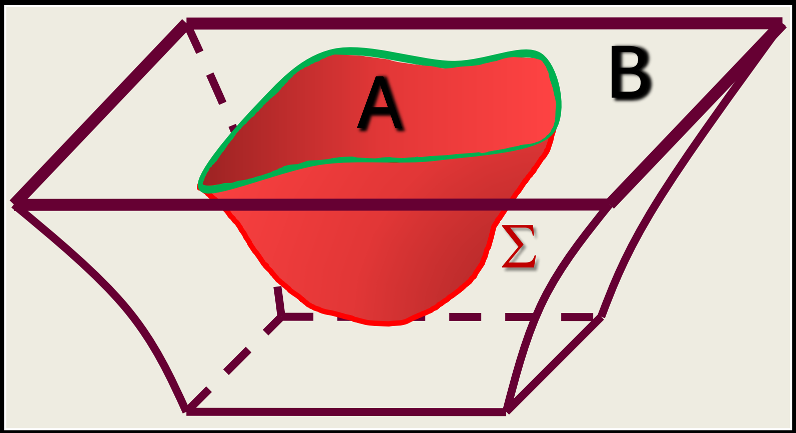
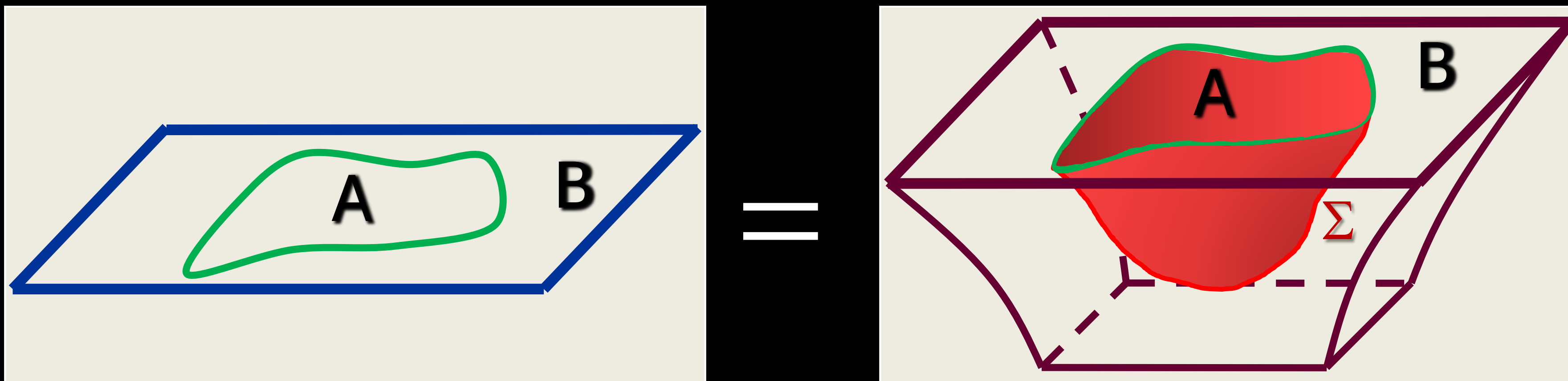


$$S(A) = \min \frac{\text{Area of } \Sigma}{4G_N}$$ (for Einstein gravity)

[Ryu,Takayanagi; Hubeny,Rangamani,Takayanagi; Lewkowycz,Maldacena; Dong,Lewkowycz,Rangamani]

Extremely useful as a tool to **COMPUTE** entanglement entropy in **STRONGLY-COUPLED QFTs**!!

Entanglement in Holography

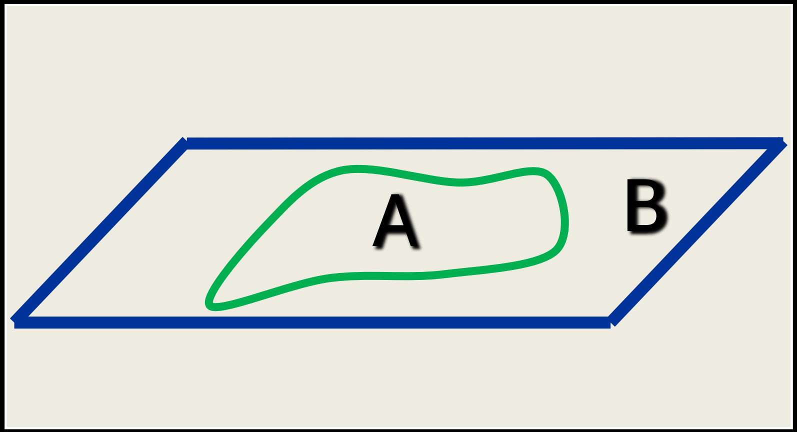


$$S(A) = \min \frac{\text{Area of } \Sigma}{4G_N}$$ (for Einstein gravity)

→

If we know the **PATTERN OF ENTANGLEMENT** of the QFT state, we know **SOME AREAS** in the gravity theory, **INFORMATION ABOUT THE BULK GEOMETRY**!!

Entanglement in Holography



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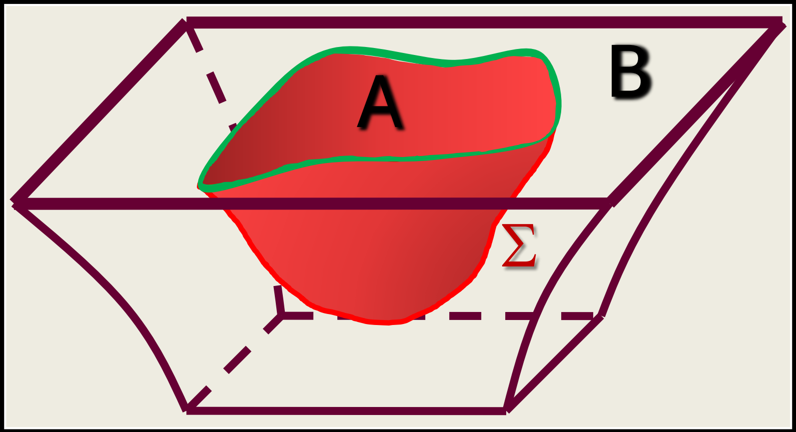

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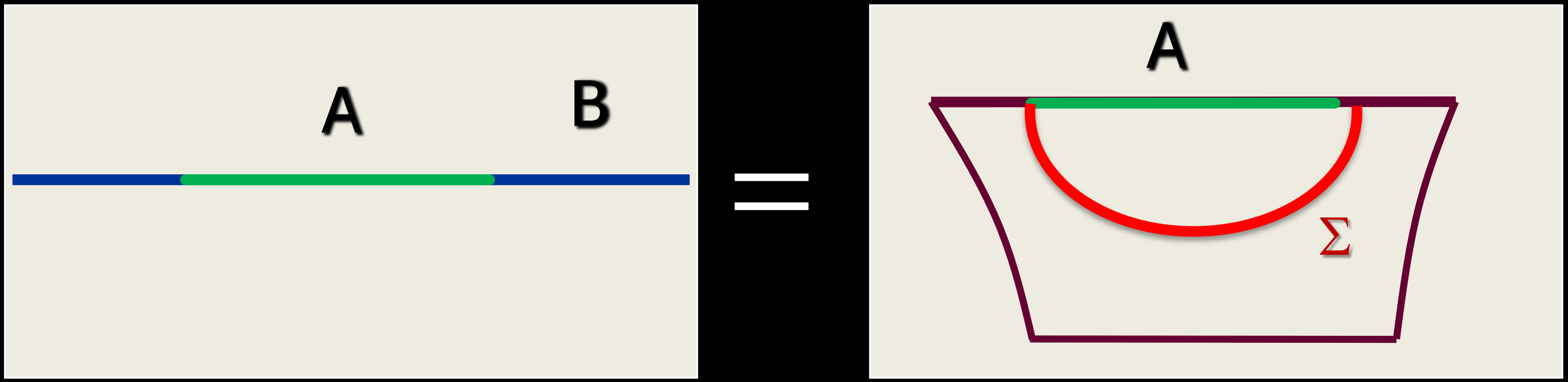
(for Einstein gravity)

If we know the **PATTERN OF ENTANGLEMENT** of the QFT state, we know **SOME AREAS** in the gravity theory

Areas of **MINIMAL** surfaces **ENDING ON THE BOUNDARY**

Entanglement in Holography



We'll work with a CFT that has only
ONE SPATIAL dimension (d=2)

The dual geometry is 2+1 dim AdS

"Minimal area surfaces", which **compute entanglement**, are then **MINIMAL LENGTH CURVES**, i.e., **GEODESICS** ending on bdry

AdS Spacetime

AdS Spacetime

Can define AdS in **GLOBAL COORDINATES**:

$$ds^2 = L^2\left(-\cosh^2 r\, d\tau^2 + dr^2 + \sinh^2 r\, d\theta^2\right)$$

$$\sinh r \equiv \tan\rho$$

$$= \frac{L^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho\, d\theta^2\right)$$

$$-\infty < \tau < \infty,$$

$$0 \le \rho < \frac{\pi}{2} \;\; (0 \le r < \infty),$$

$$0 \le \theta < 2\pi$$

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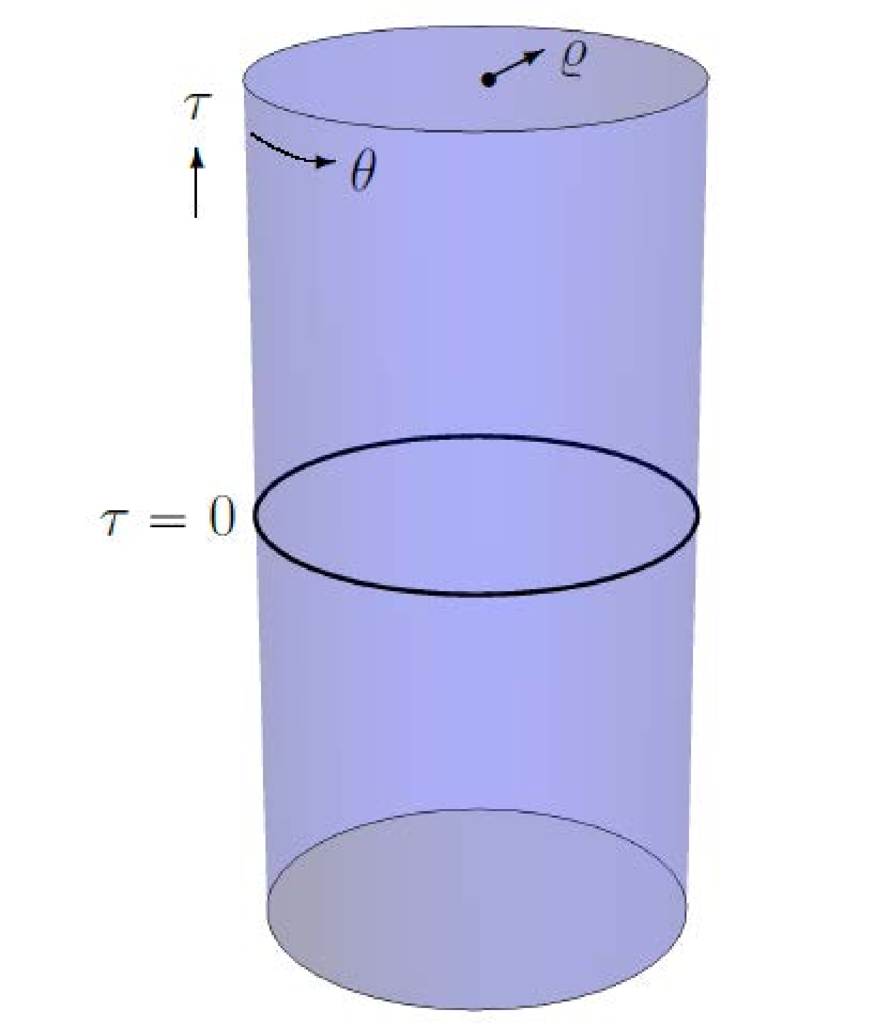

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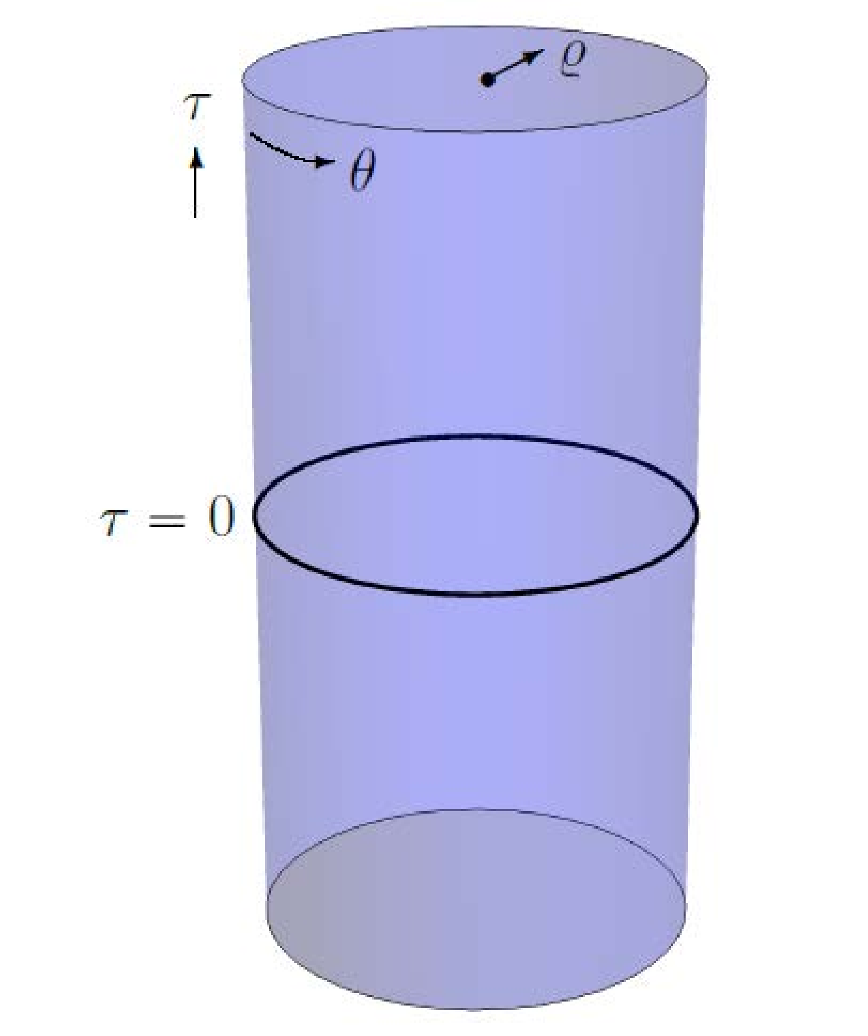
Can define AdS in **GLOBAL COORDINATES**:

$$ds^2 = \frac{L^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho\, d\theta^2\right)$$

Dual CFT then lives on copy of the AdS boundary,

$$ds^2 = -d\tau^2 + d\theta^2$$

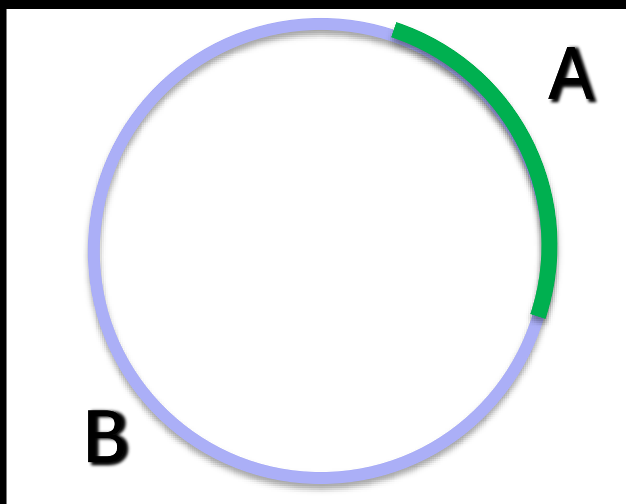
$$\mathrm{R} \times \mathrm{S}^1$$



AdS Spacetime

Can define AdS in **GLOBAL COORDINATES**

At **CONSTANT TIME**:



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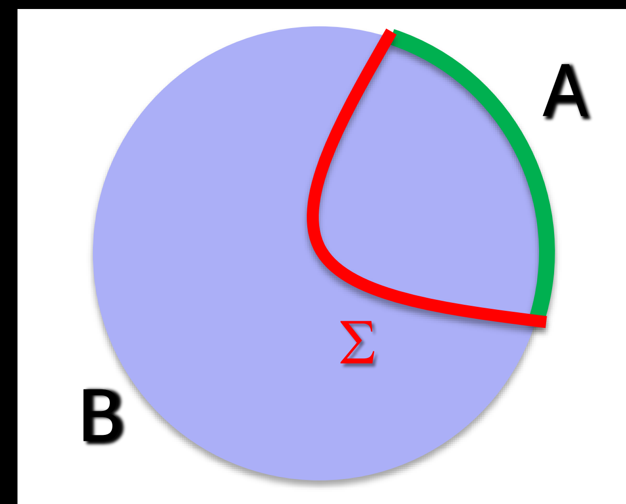


CFT on circle

$$ds^2 = d\theta^2$$

GRAVITY THEORY on hyperbolic disk (AdS)

$$ds^2 = \frac{L^2}{\cos^2 \rho}\left(d\rho^2 + \sin^2 \rho \, d\theta^2\right)$$

AdS Spacetime

Can define AdS in **POINCARÉ COORDINATES**:

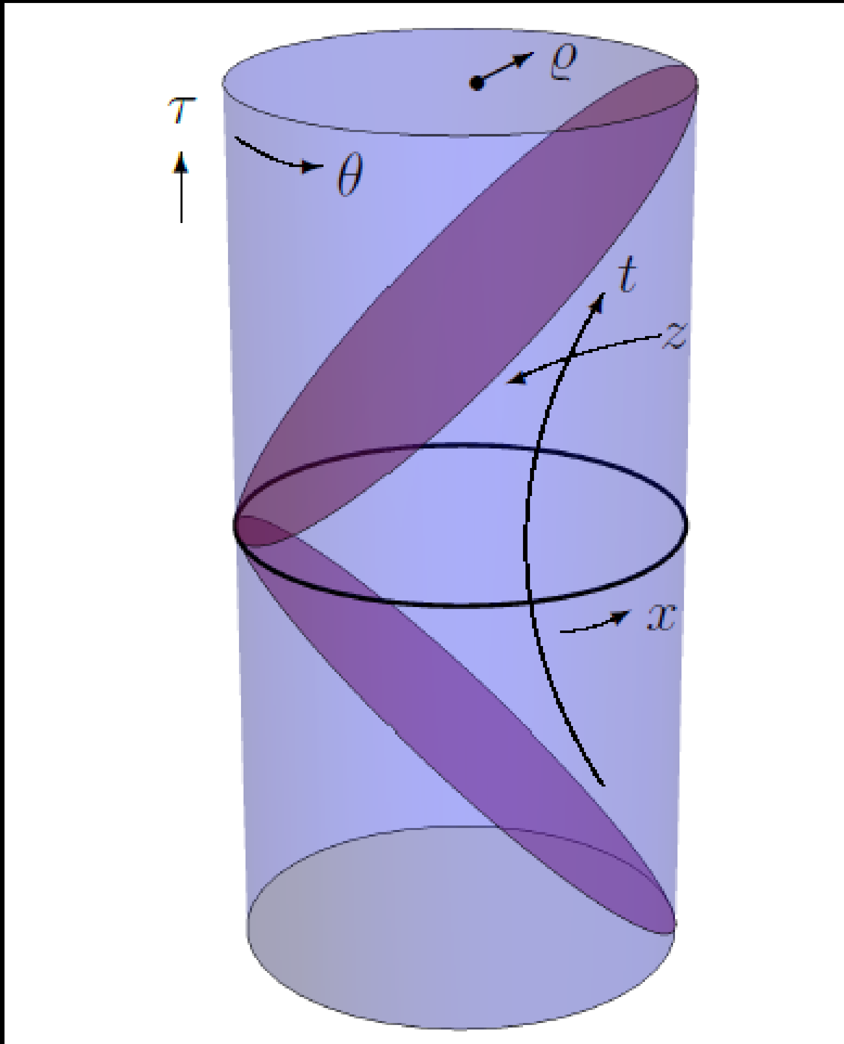
$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dx^2 + dz^2\right)$$

$$-\infty < t, x < \infty, 0 < z < \infty$$

$$t = \frac{L \sin\tau}{\cos\tau + \sin\rho\cos\theta},$$

$$x = \frac{L \sin\theta \sin\rho}{\cos\tau + \sin\rho\cos\theta},$$

$$z = \frac{L\cos\rho}{\cos\tau + \sin\rho\cos\theta}$$



AdS Spacetime

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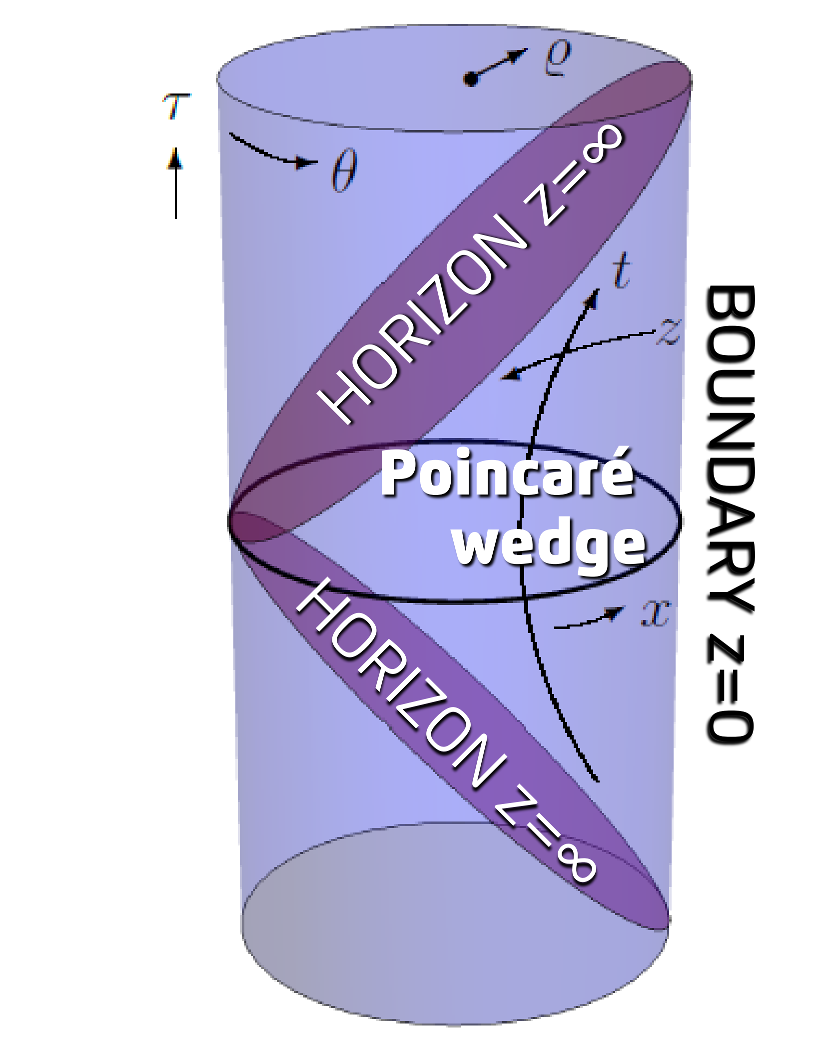
$$-\infty < t, x < \infty, 0 < z < \infty$$

These coordinates, associated with uniformly **ACCELERATED** observers with proper acceleration $1/L$, **ONLY COVER A FINITE REGION within AdS**



AdS Spacetime

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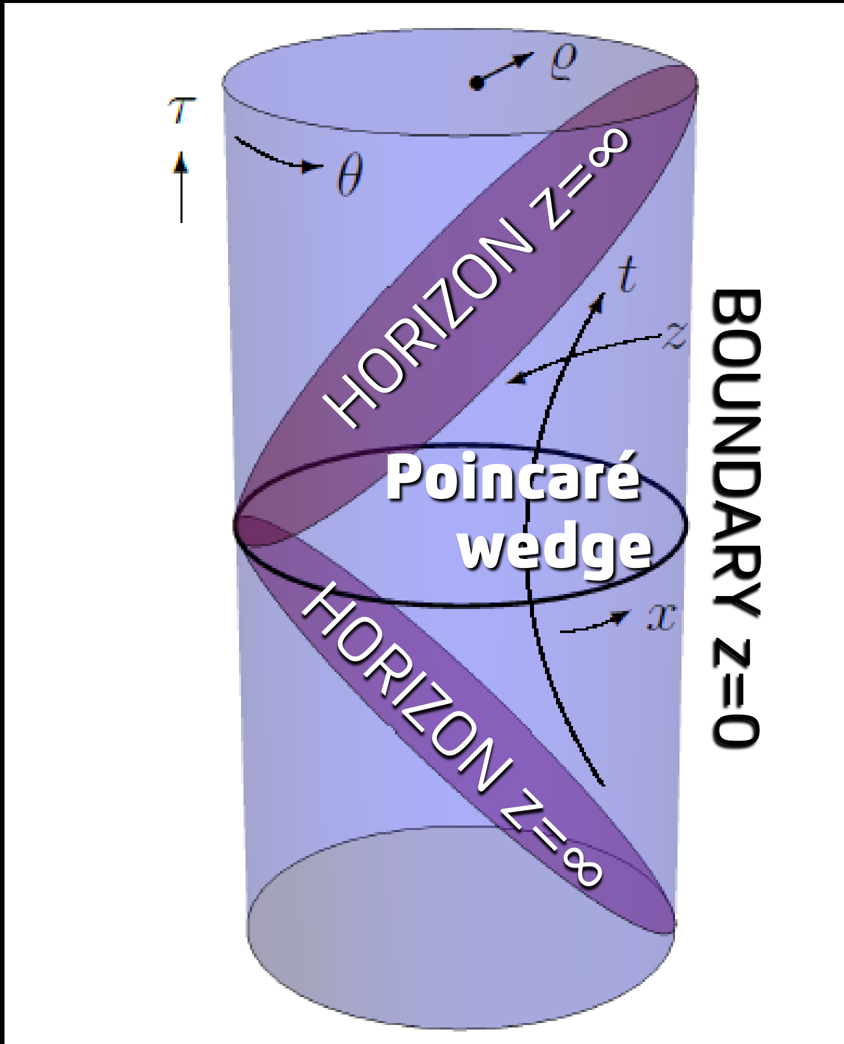
$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dx^2 + dz^2\right)$$

$$-\infty < t, x < \infty, 0 < z < \infty$$

Dual CFT then lives on a copy of the boundary of the wedge,

$$ds^2 = -dt^2 + dx^2$$

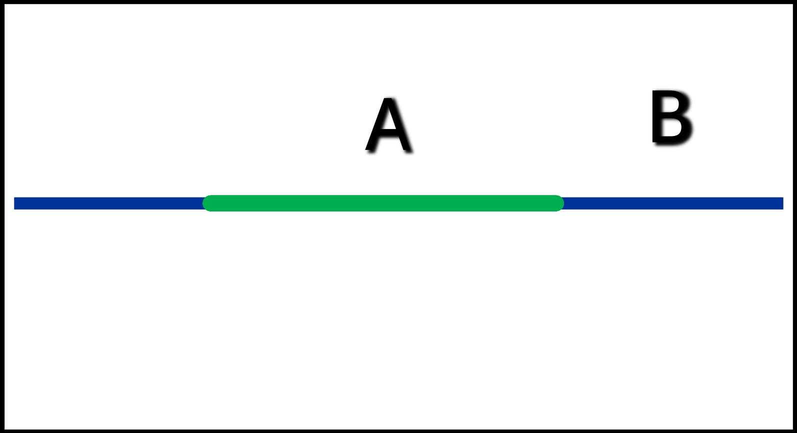
$$\mathrm{R}^{1,1}$$



AdS Spacetime

Can define AdS in **POINCARÉ COORDINATES**

At **CONSTANT TIME**:



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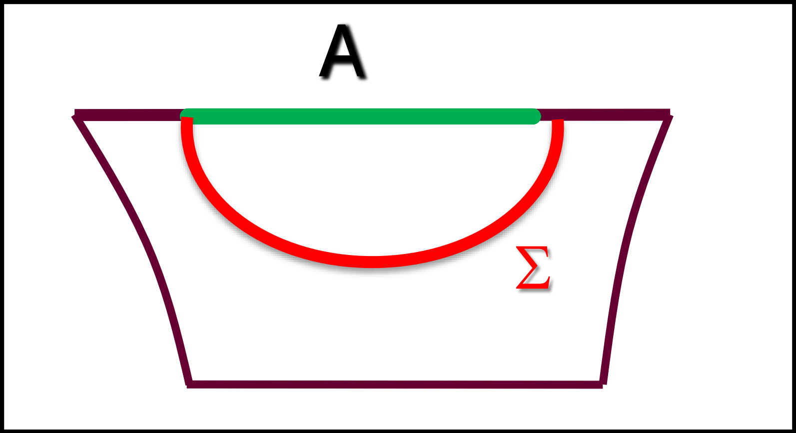


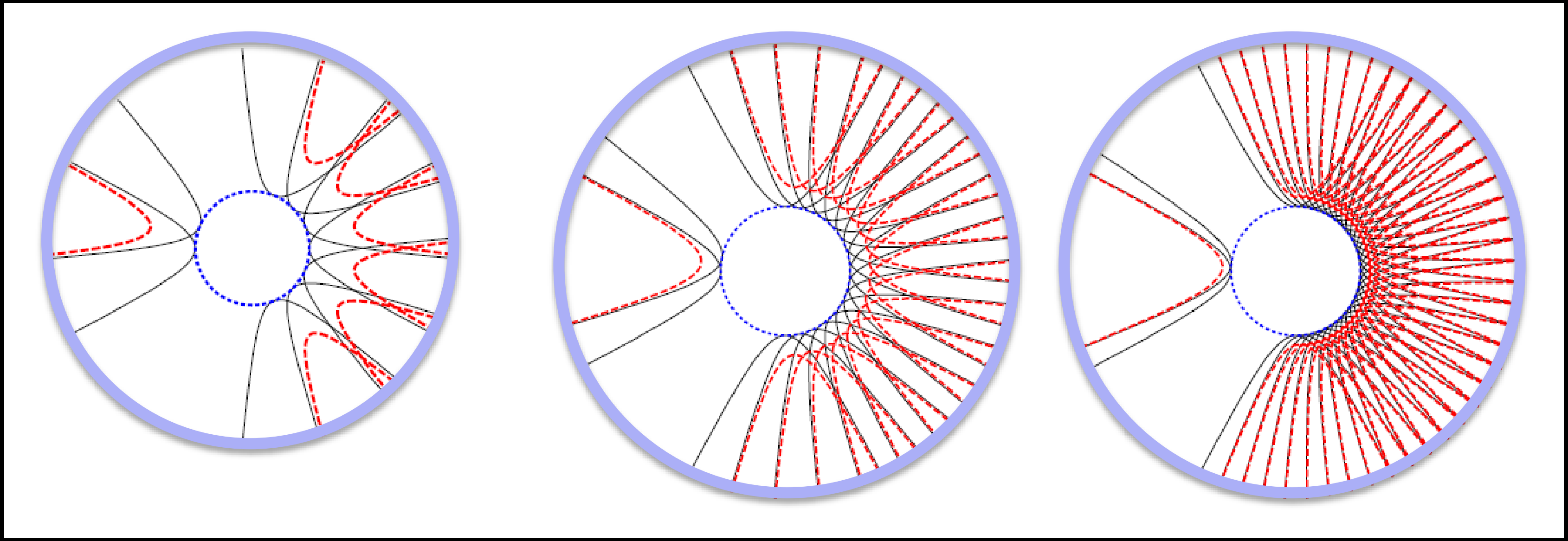
CFT on line

$$ds^2 = dx^2$$

GRAVITY THEORY on hyperbolic plane (AdS)

$$ds^2 = \frac{L^2}{z^2}\left(dx^2 + dz^2\right)$$

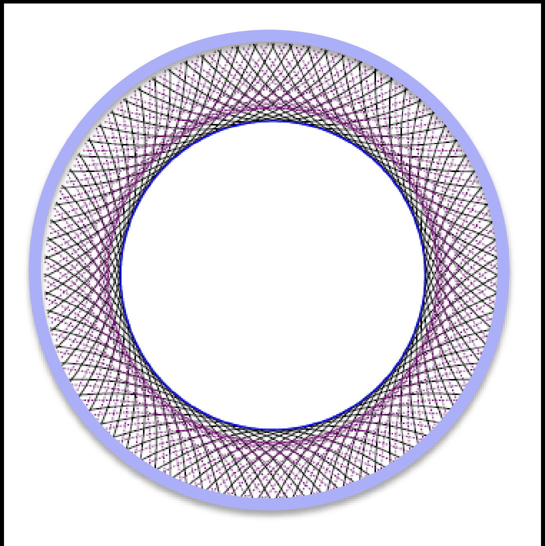
HOLE-OGRAPHY

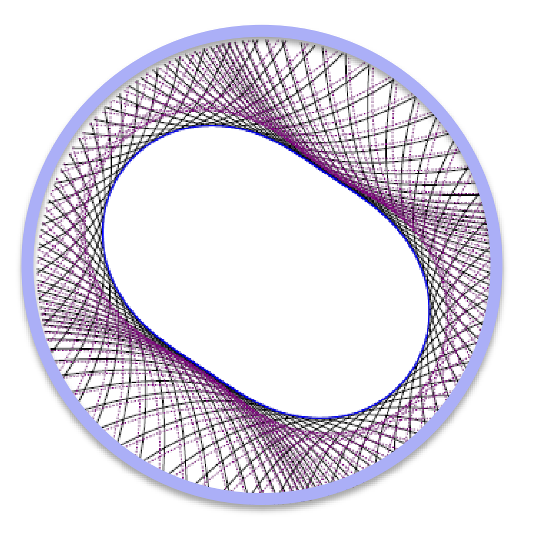


We can **reconstruct ANY SPACELIKE CURVE** by adding and subtracting geodesics that are **TANGENT** to the curve

[Balasubramanian,Chowdhury,Czech,de Boer,Heller Myers,Rao,Sugishita; Czech,Dong,Sully; Czech,Lamprou,McCandlish,Sully; Headrick,Myers,Wien]

HOLE-OGRAPHY





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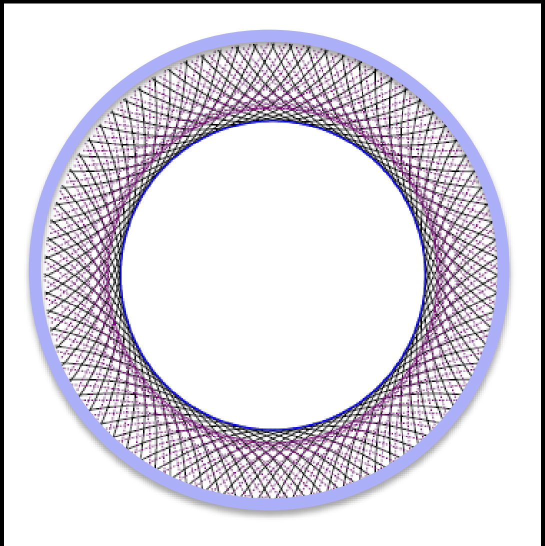
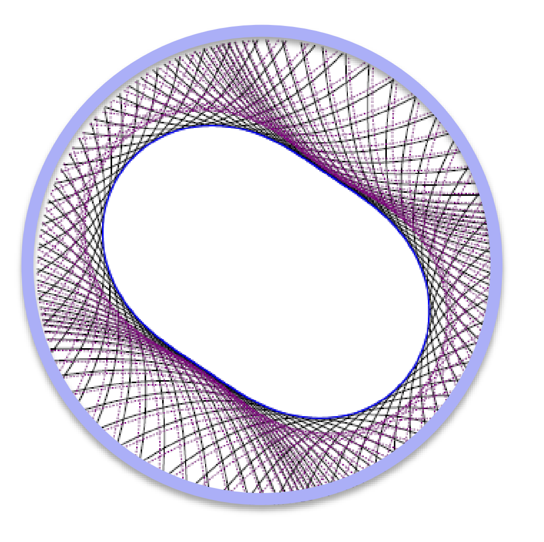



We can **reconstruct ANY SPACELIKE CURVE** by adding and subtracting geodesics that are **TANGENT** to the curve

Originally phrased in terms of the HOLE carved out by the bulk curve: `**HOLE-OGRAPHY**'

ANY curve, not necessarily reaching boundary!

HOLE-OGRAPHY

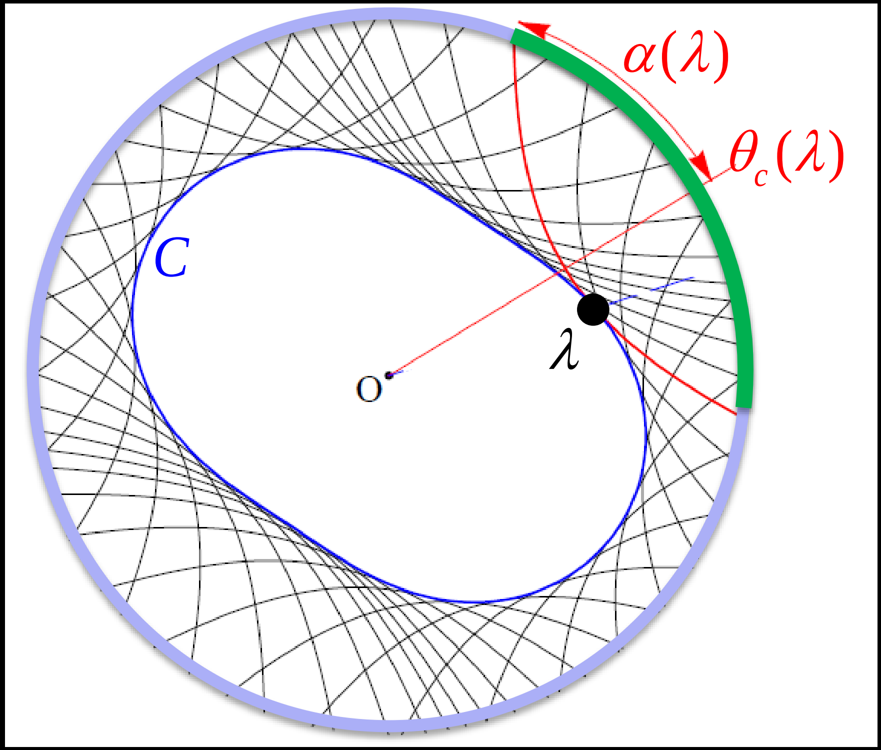
In more detail:

1) Each bulk curve C gets mapped to a **FAMILY OF INTERVALS** in the CFT, specified by center and opening angle

$$\theta_c(\lambda), \alpha(\lambda)$$

or by ENDPOINTS

$$\theta_\pm(\lambda) \equiv \theta_c(\lambda) \pm \alpha(\lambda)$$



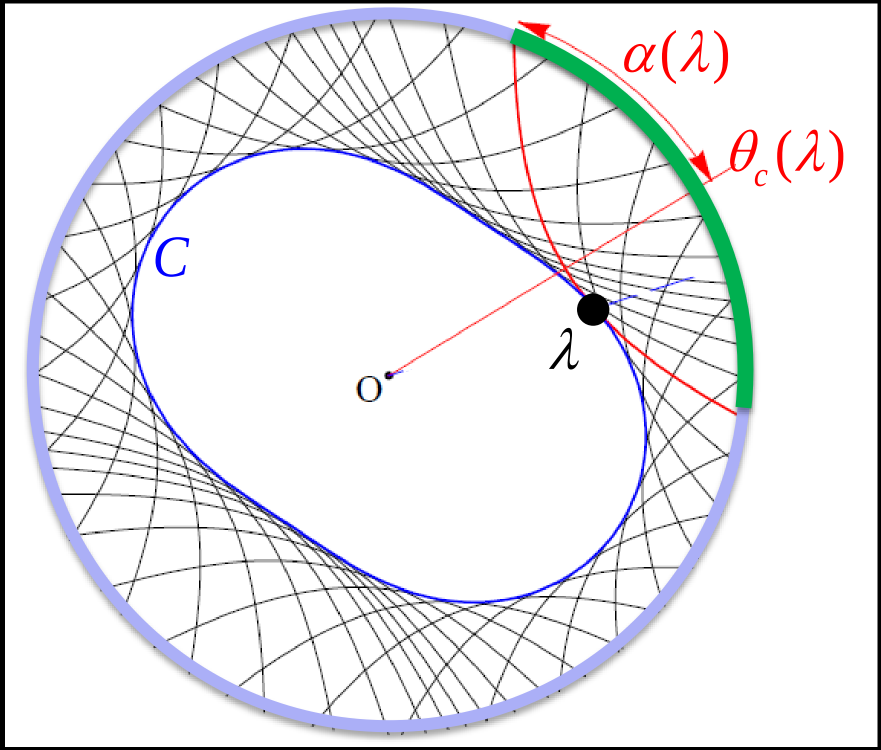
HOLE-OGRAPHY

In more detail:

1) Each bulk curve C gets mapped to a **FAMILY OF INTERVALS** in the CFT, specified by ENDPOINTS

$$\theta_{\pm}(\lambda) \equiv \theta_c(\lambda) \pm \alpha(\lambda)$$

There is an **ENTANGLEMENT ENTROPY** associated to each interval:



$$S(\theta_-, \theta_+) = \frac{L}{2G_N} \ln\left[\sin\left(\theta_+ - \theta_-\right)/2\delta\right]$$

δ is a **UV cutoff**

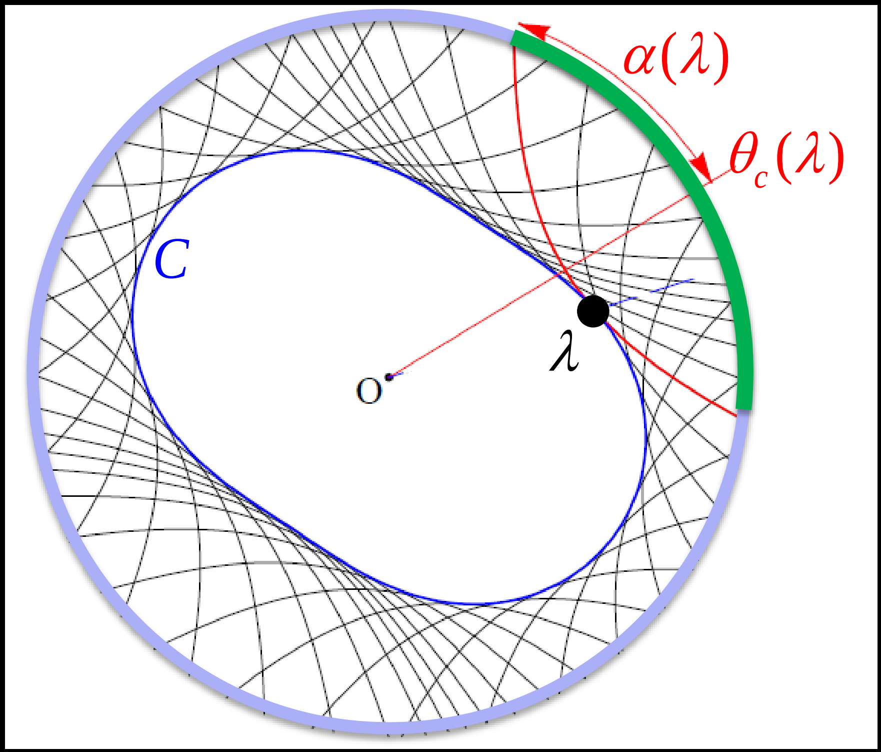
HOLE-OGRAPHY

In more detail:

1) Each bulk curve C gets mapped to a **FAMILY OF INTERVALS** in the CFT, specified by ENDPOINTS

$$\theta_{\pm}(\lambda) \equiv \theta_c(\lambda) \pm \alpha(\lambda)$$

2) The "**AREA**" (length) of C is computed by the family's **DIFFERENTIAL ENTROPY**



$$\boxed{E \equiv \oint d\lambda \left(\frac{\partial S(\theta_-(\lambda), \theta_+(\bar{\lambda}))}{\partial \bar{\lambda}} \right)\Bigg|_{\bar{\lambda}=\lambda} = \frac{A}{4G_N}}$$

[Balasubramanian et al.; Myers et al.; Headrick et al.]

HOLE-OGRAPHY

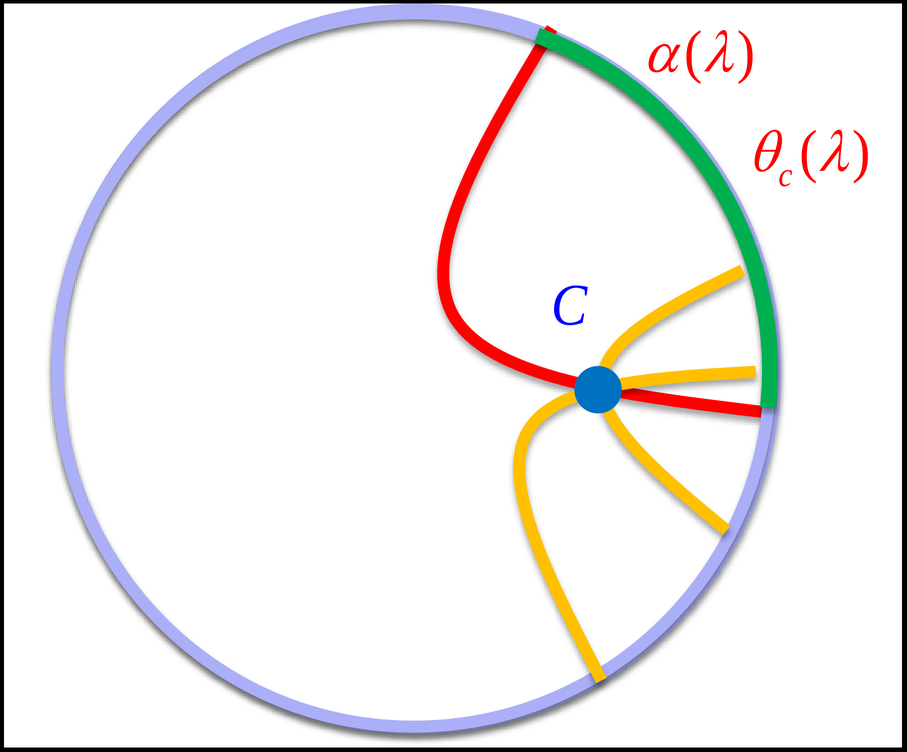
Once we have a closed curve, we can shrink it down to a **BULK POINT**

Intervals in CFT

$$\theta_{\pm}(\lambda) \equiv \theta_c(\lambda) \pm \alpha(\lambda)$$

are then associated with all geodesics **PASSING THROUGH** the point

[Czech,Lamprou]

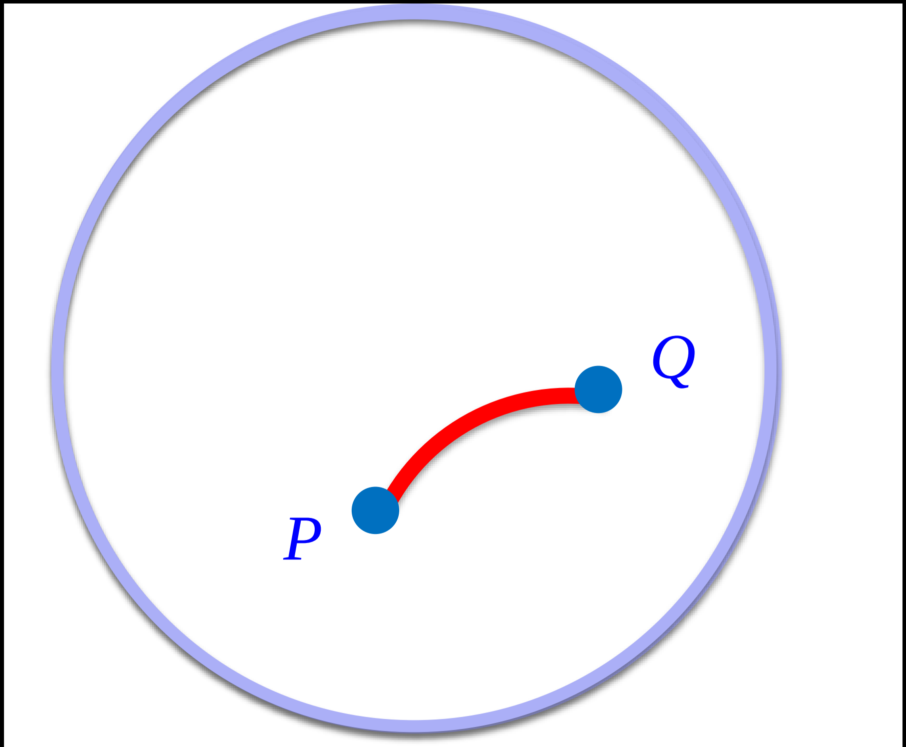


HOLE-OGRAPHY

Once we have a closed curve, we can shrink it down to a **BULK POINT**

It is possible to compute the **DISTANCE** between two bulk points *P* and *Q* in terms of the corresponding **DIFFERENTIAL ENTROPIES**

[Czech,Lamprou]

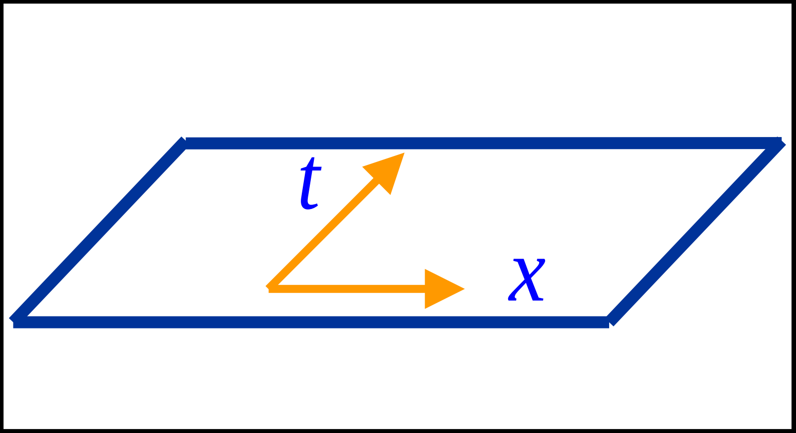


So the most basic ingredients of the (spatial) geometry, **POINTS** and **DISTANCES**, can be **recovered purely from the pattern of entanglement in the CFT state**!!

PART 1

Hole-ography in Poincaré

In this talk we are interested in examining these same issues in **POINCARÉ AdS**



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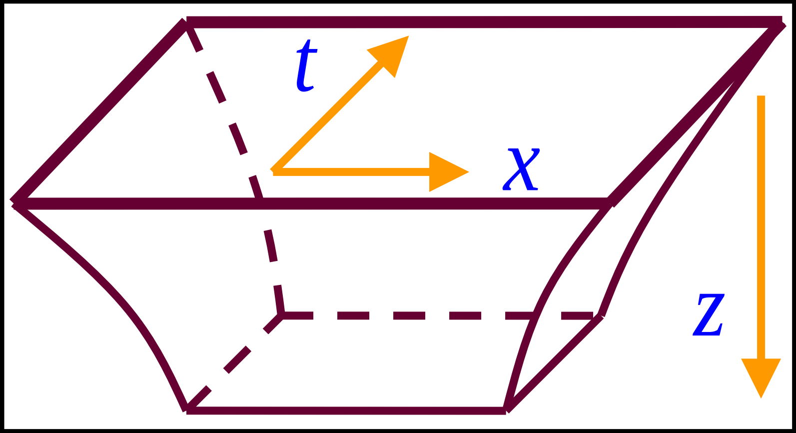


CFT on Minkowski 1+1

$$ds^2 = -dt^2 + dx^2$$

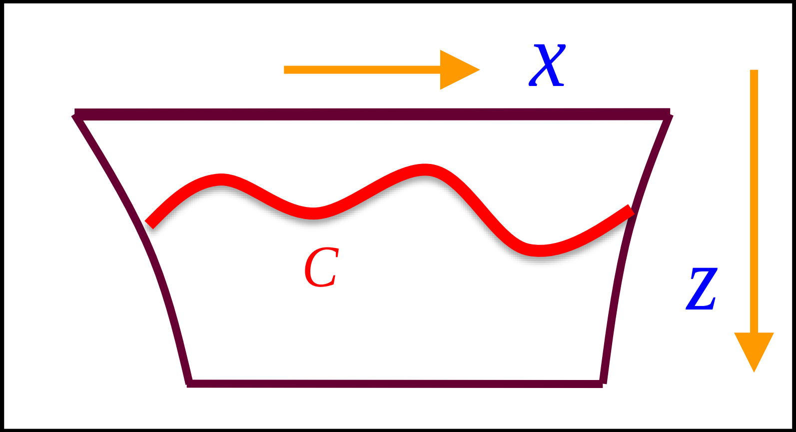
GRAVITY on Poincaré AdS

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dx^2 + dz^2\right)$$

Hole-ography in Poincaré

In this talk we are interested in examining these same issues in **POINCARÉ AdS**

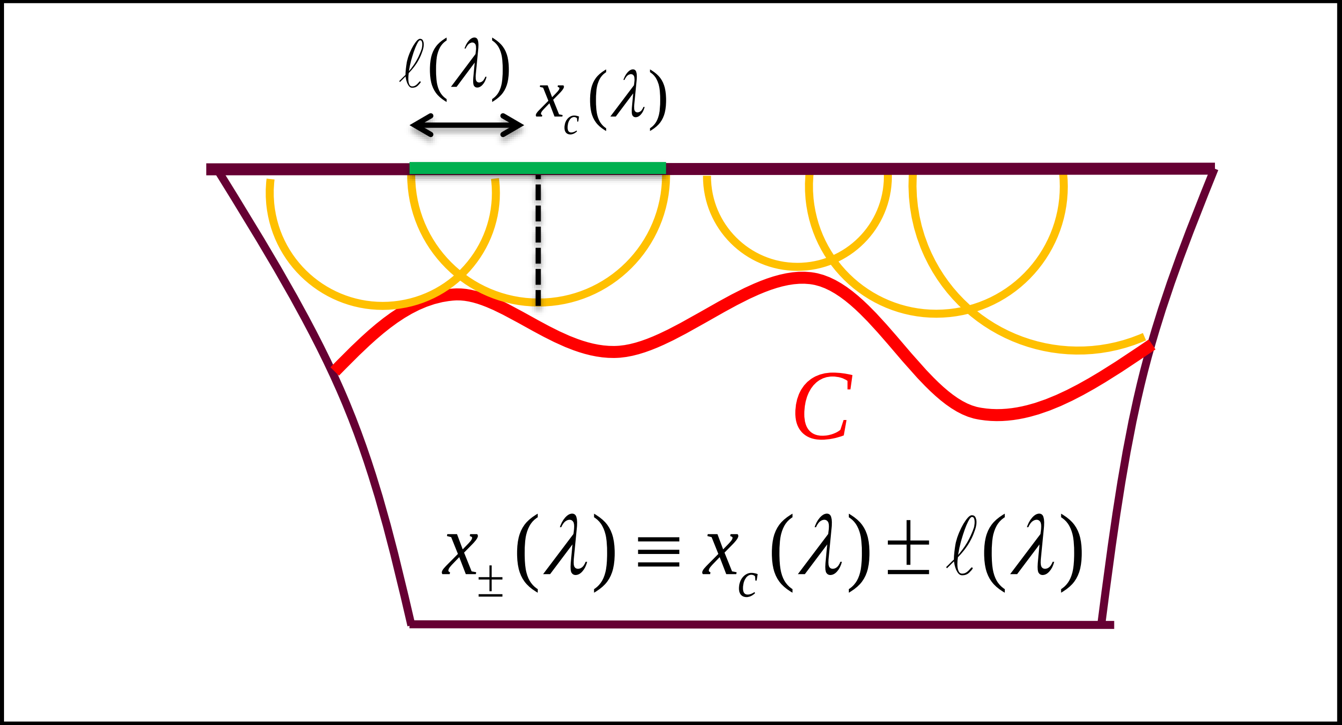
Hole-ography in Poincaré has been studied before, both at fixed time [Myers,Rao,Sugishita] and at varying time [Headrick,Myers,Wien]



But only for curves C that are **INFINITELY EXTENDED** and satisfy periodicity condition at spatial infinity

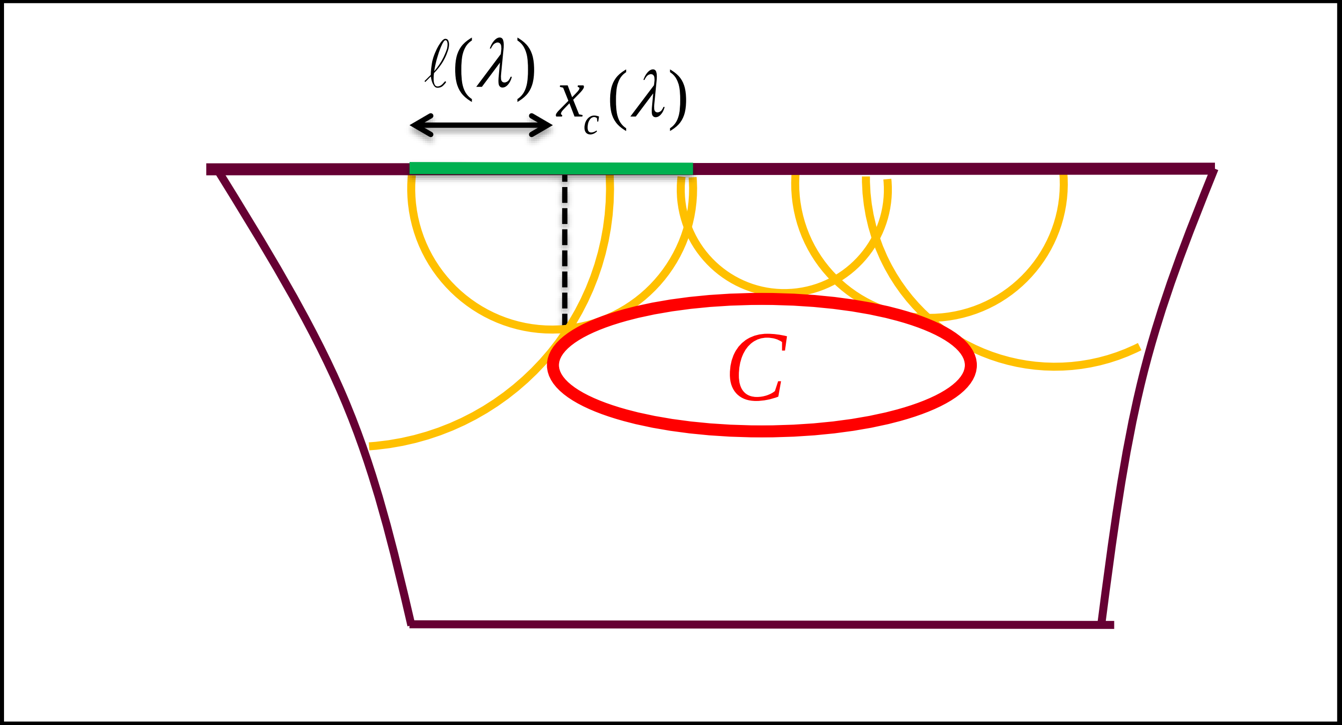
Hole-ography in Poincaré

For this type of curves, easy to get tangent geodesics:



Hole-ography in Poincaré

But how can we describe a **CLOSED CURVE**? Need for point!



Hole-ography in Poincaré

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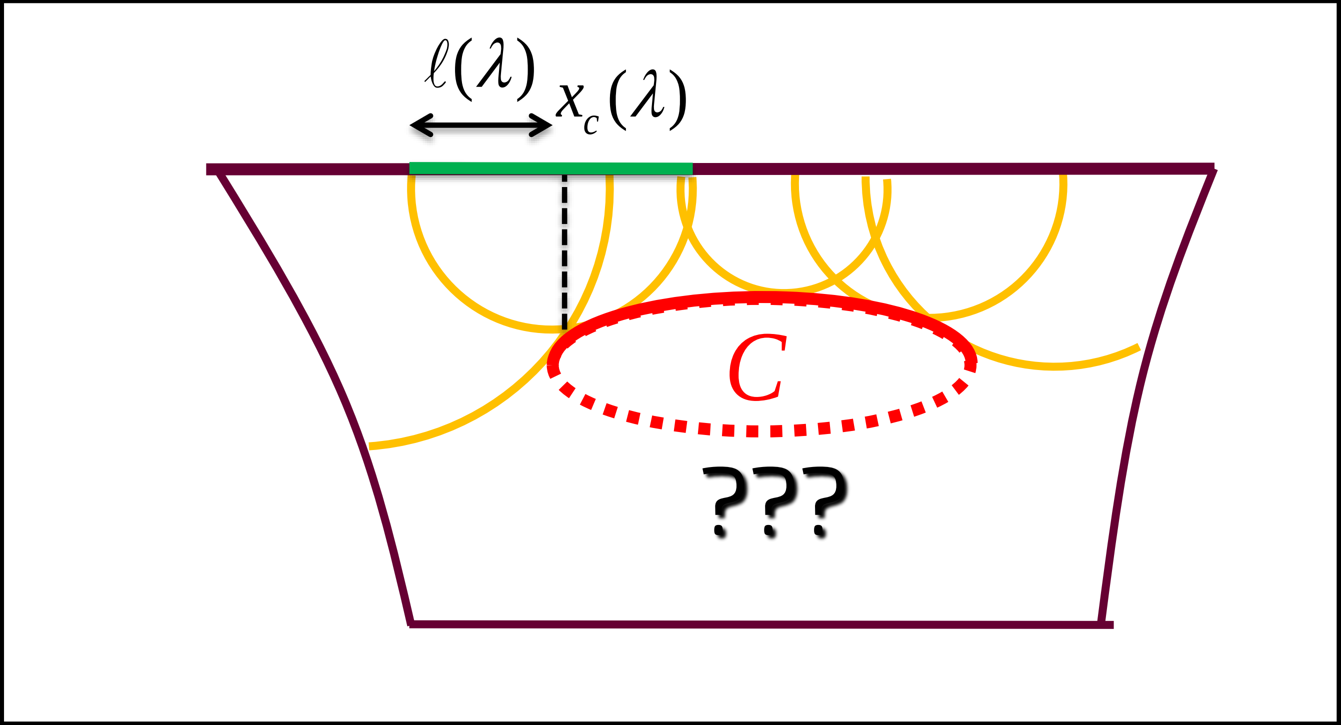
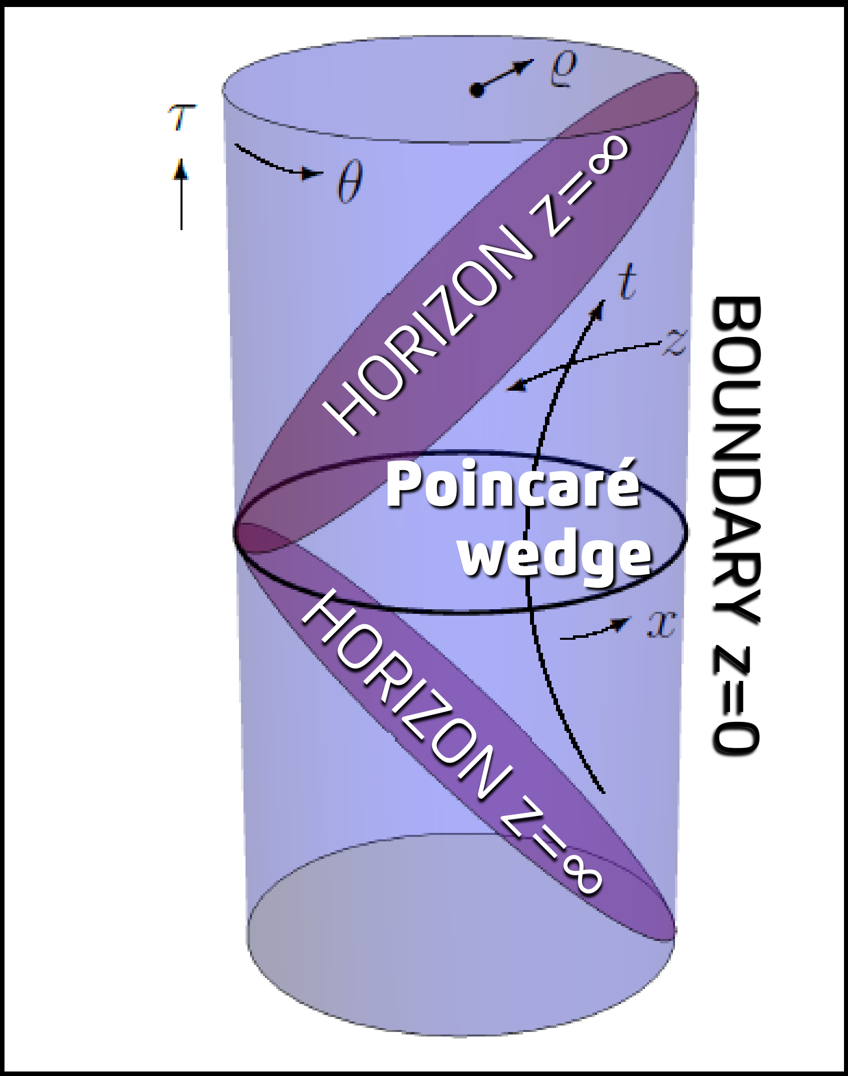


We seem to be missing geodesics…

…which is indeed possible, since Poincaré wedge **ONLY COVERS PART OF AdS!**

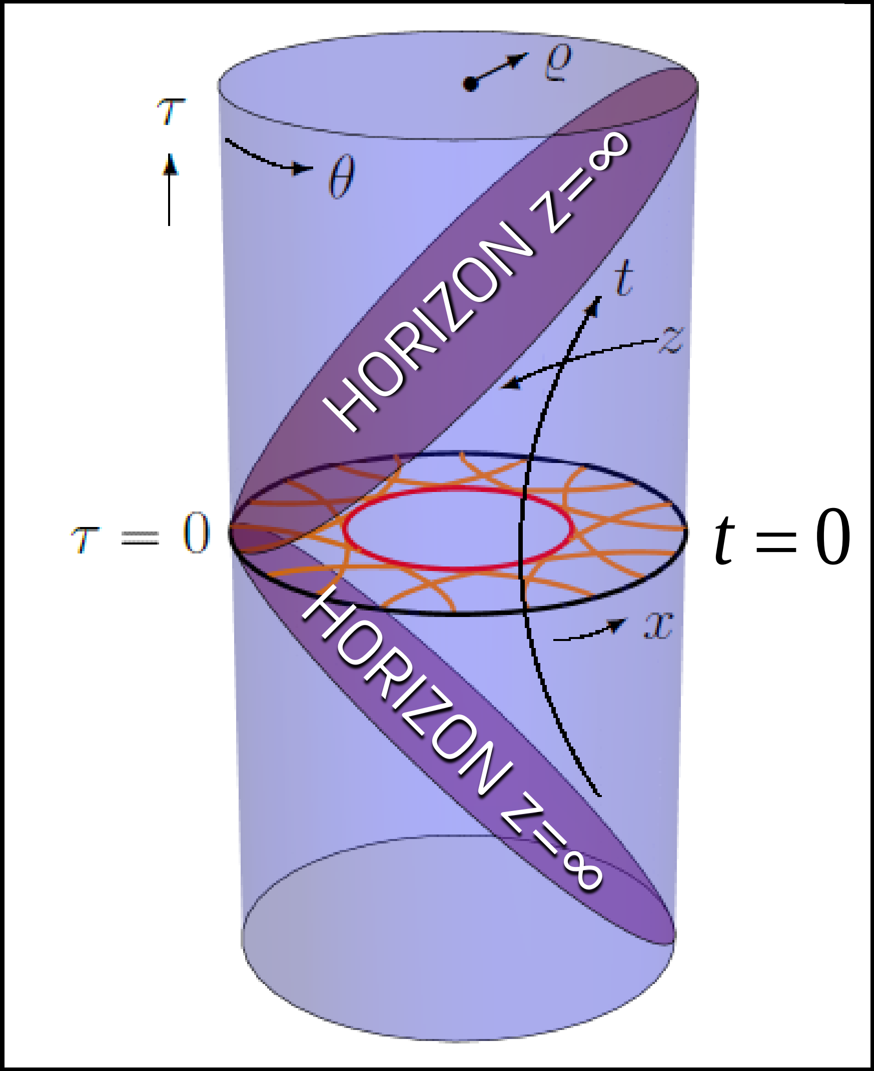
Hole-ography in Poincaré

Recall relation beween Poincaré and global AdS:



Hole-ography in Poincaré

Recall relation beween Poincaré and global AdS:



Slice of constant Poincaré time $t = 0$ coincides with slice of constant global time $\tau = 0$

So at $t = 0$ we CANNOT be missing any geodesics

And by *t*-independence of

$$ds^2 = \frac{L^2}{z^2}\left(-dt^2 + dx^2 + dz^2\right)$$

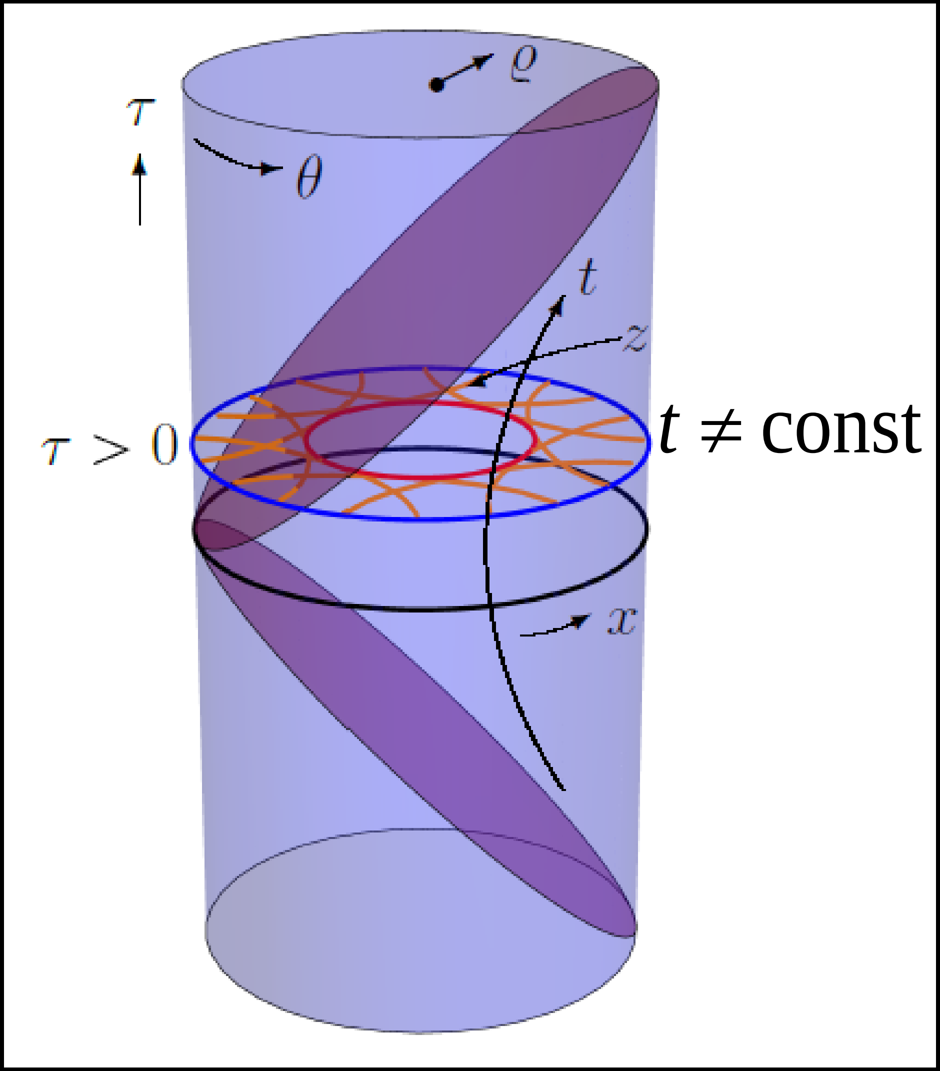
won't be missing geodesics on ANY fixed-*t* slice, either

Hole-ography in Poincaré

Recall relation beween Poincaré and global AdS:

On the other hand, slice of constant global time $\tau \neq 0$ (corresponding to VARYING Poincaré time) is partly **OUTSIDE** of the Poincaré wedge, so on this slice we have **curves that are NOT FULLY RECONSTRUCTIBLE within Poincaré AdS!!**

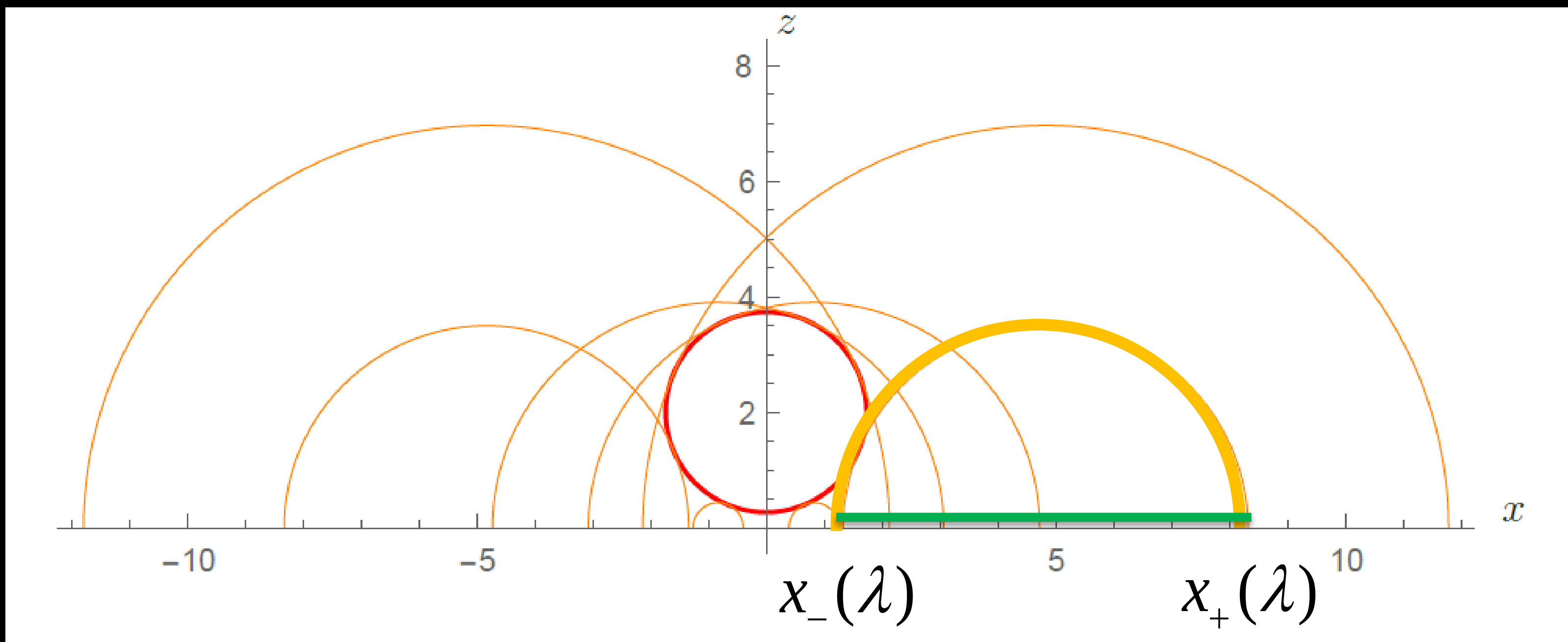
This is a **CHALLENGE** to hole-ography



Poincaré at Fixed Time

We do find some novelties:

1) CLOSED curves are described by **a family of CFT intervals that covers the *x*-axis AT LEAST TWICE**



Upper portion is covered on second pass: after one of the endpoints crosses through $\pm\infty$

Poincaré at Fixed Time

We do find some novelties:

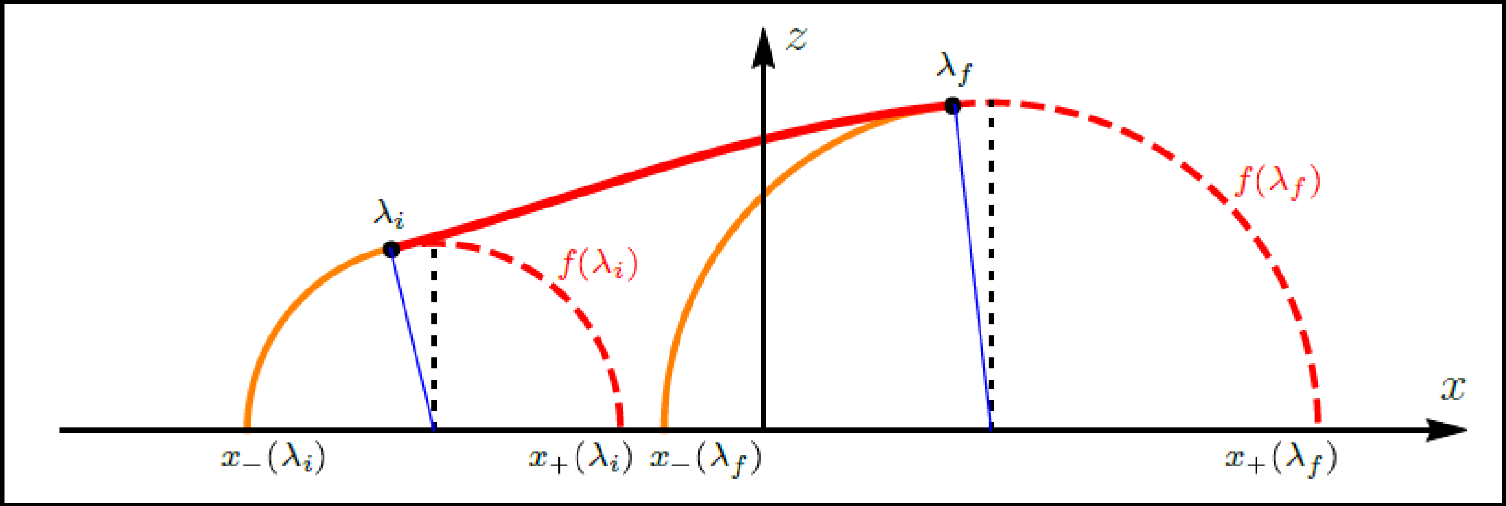
1) CLOSED curves are described by **a family of CFT intervals that covers the *x*-axis AT LEAST TWICE**

Can verify that **DIFFERENTIAL ENTROPY** (measured henceforth in units of $1/4G_N$) correctly **REPRODUCES LENGTH**

$$\boxed{E \equiv \oint d\lambda \left(\frac{\partial S(x_-(\lambda), x_+(\bar{\lambda}))}{\partial \bar{\lambda}} \right)\Bigg|_{\bar{\lambda}=\lambda} = A}$$

Poincaré at Fixed Time

2) For OPEN curves, besides E need a **BOUNDARY TERM** (= total-derivative), that has a clear interpretation on BOTH sides of the duality (in CFT, entanglement)



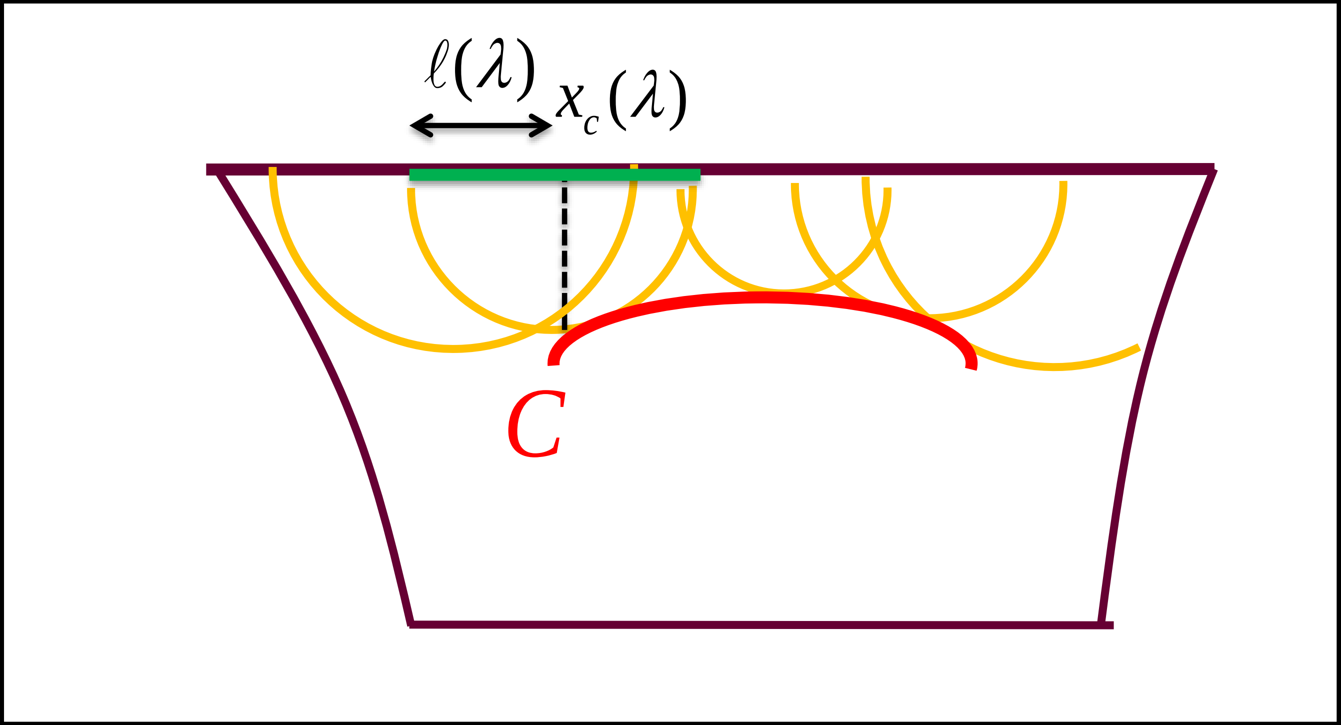
$$A = E - f(\lambda_f) + f(\lambda_i)$$

$$= L\int_{\lambda_i}^{\lambda_f} d\lambda \left(\frac{x_c'}{\ell} + \frac{\ell' x_c'' - \ell'' x_c'}{x_c'^2 - \ell'^2} \right) \equiv \mathcal{E}$$

'RENORMALIZED' DIFFERENTIAL ENTROPY

Poincaré at Fixed Time

3) Can describe **BULK POINTS** by shrinking either **CLOSED** curves or **OPEN** curves that become VERTICAL at their edges



Poincaré at Fixed Time

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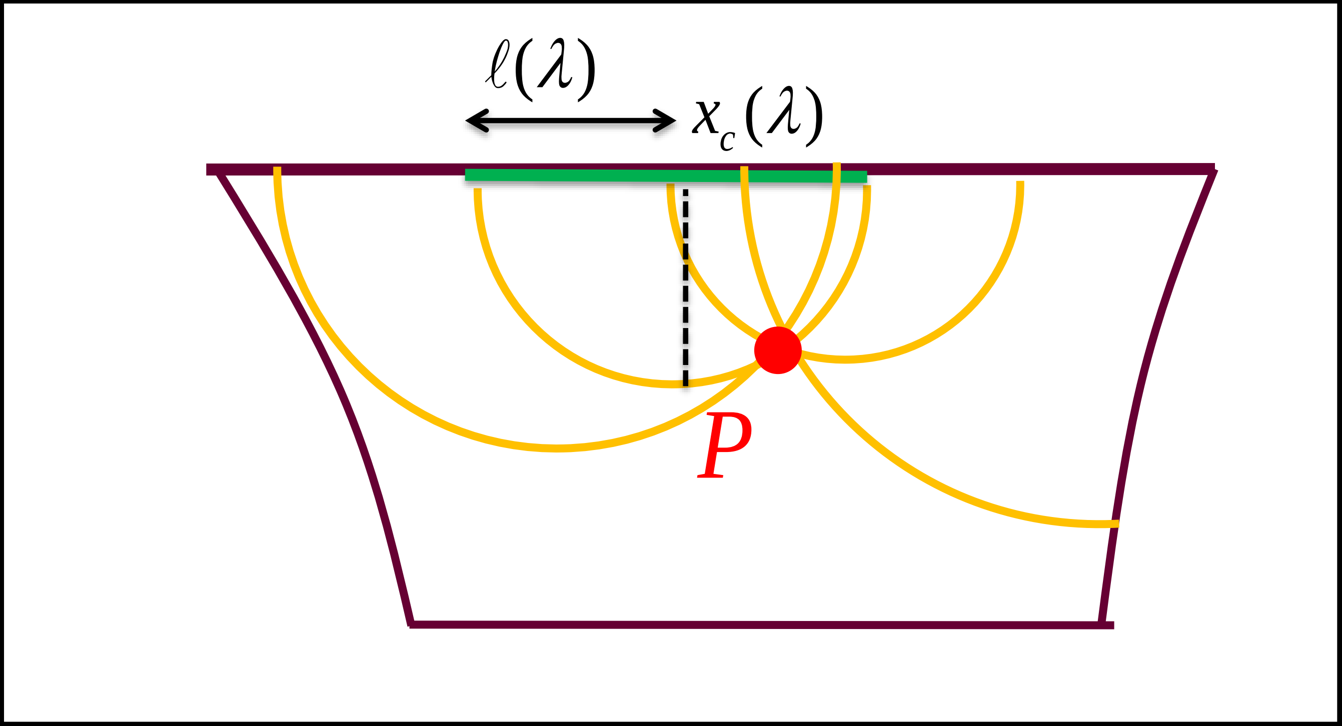
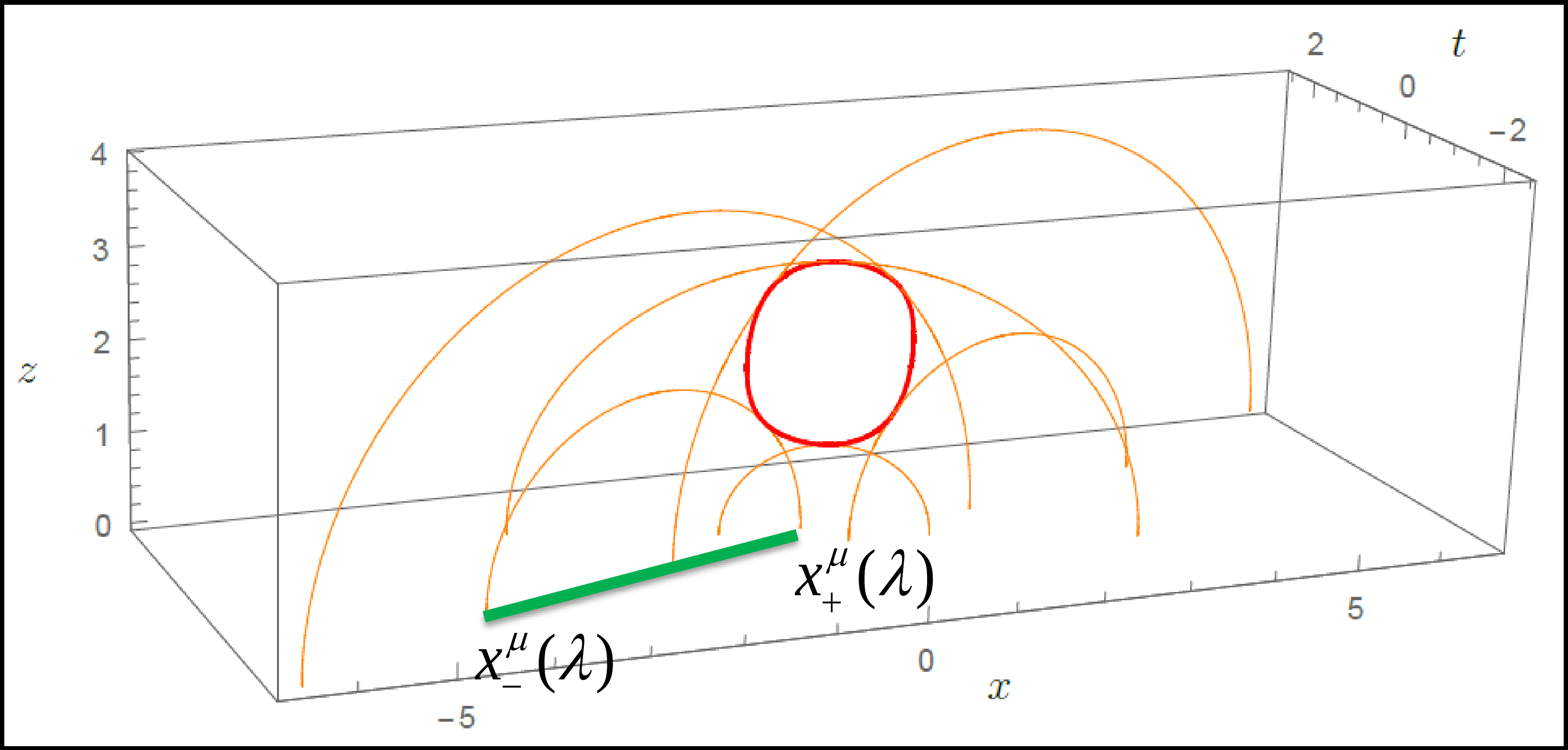


Poincaré at Varying Time

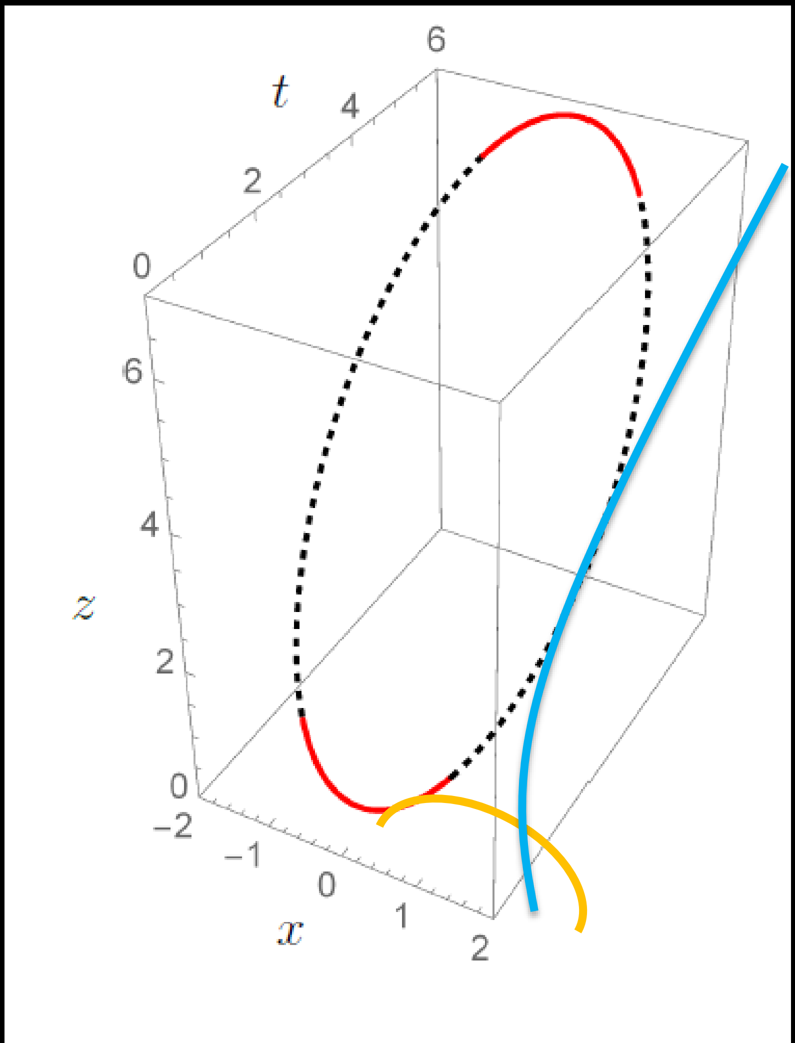
For CLOSED or OPEN curves, can find **TANGENT** geodesics



$$x^\mu_\pm(\lambda) = x^\mu_c(\lambda) \pm \ell^\mu(\lambda)$$

Failure of Reconstruction

But generically, curves contain segments whose **tangent geodesics DO NOT reach boundary of Poincaré wedge**



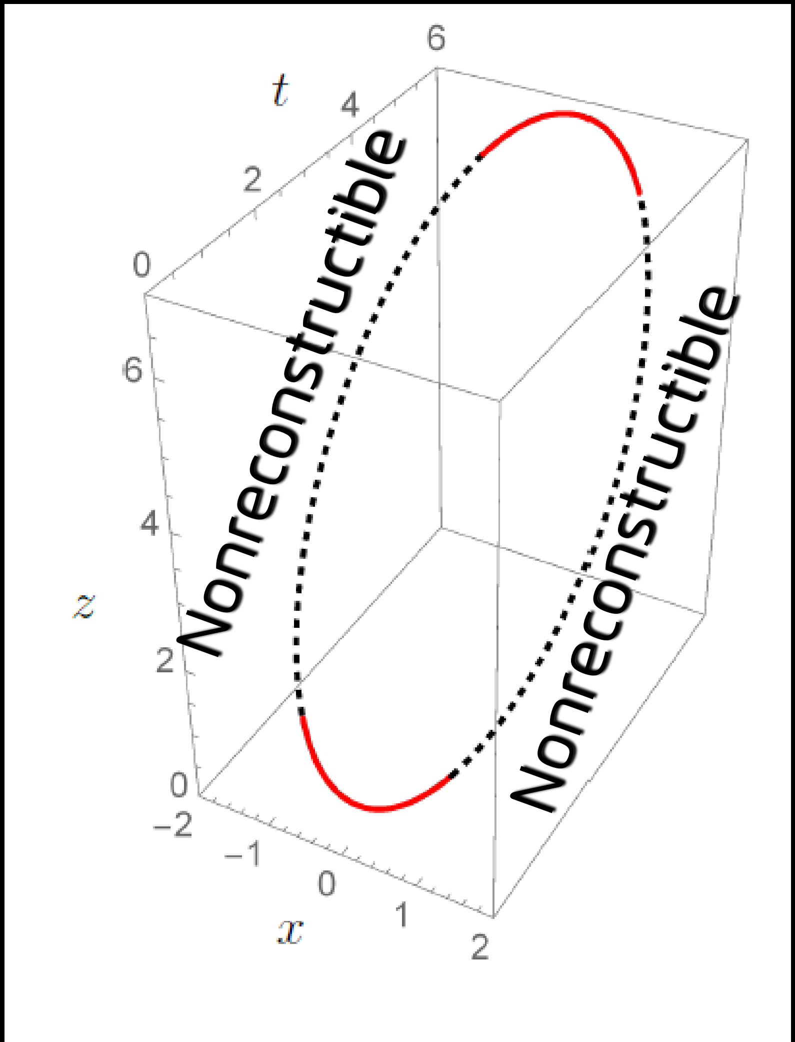
E.g., this oval tilted in the time direction (while remaining SPACELIKE)

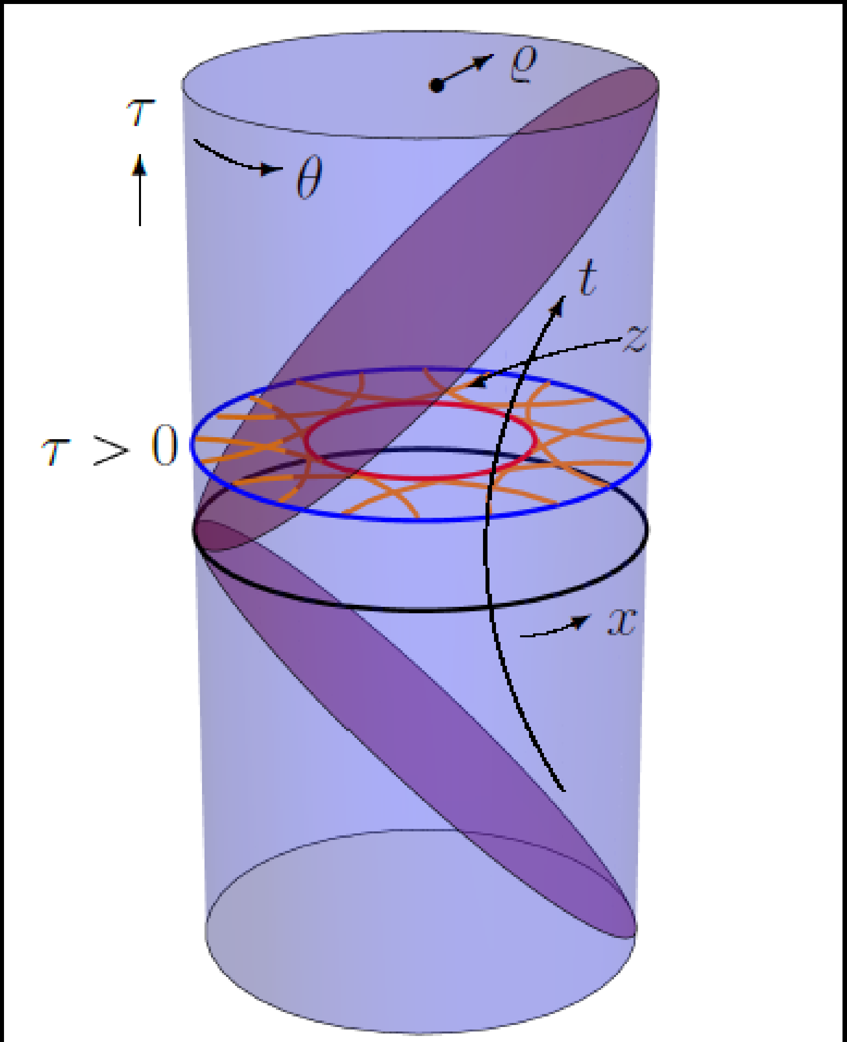
Geodesics tangent to dotted black portions **ARE NOT associated to entanglement**

In fact, this is precisely the circle at constant GLOBAL time $\tau > 0$ that we discussed before…

Failure of Reconstruction

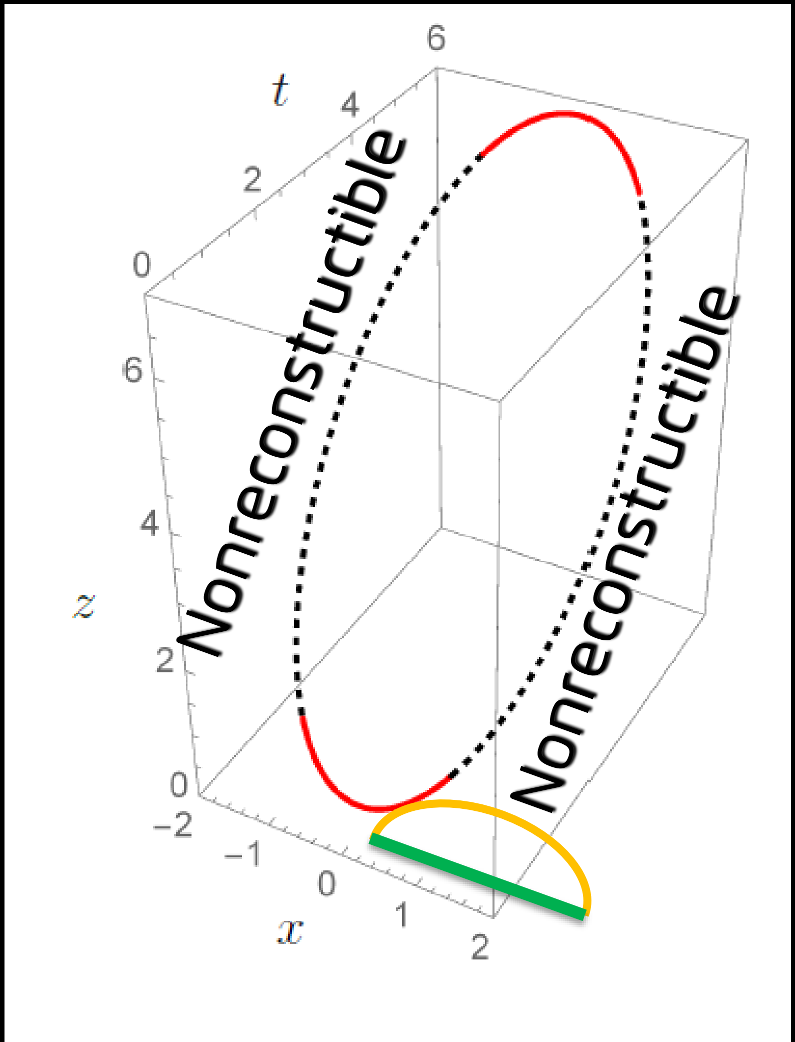
But generically, curves contain segments whose **tangent geodesics** **DO NOT reach boundary of Poincaré wedge**





Failure of Reconstruction

Tangent vector to curve is $\left(t'(\lambda), x'(\lambda), z'(\lambda)\right)$, and is **SPACELIKE**, $-t'^2 + x'^2 + z'^2 > 0$.



The **CONDITION FOR RECONSTRUCTIBLITY** is

$$\boxed{-t'^2 + x'^2 > 0}$$

Segments that violate this condition **CANNOT BE RECONSTRUCTED using standard hole-ography**

Null Vector Alignment

Given a curve with **TANGENT VECTOR** $u \equiv \left(t'(\lambda), x'(\lambda), z'(\lambda)\right)$,

$$E = \oint d\lambda \sqrt{\frac{L^2}{z^2}\left(-t'^2 + x'^2 + z'^2\right)}$$

$$= \oint d\lambda \sqrt{g_{mn} u^m u^n} = A$$

Agreement E=A is maintained if we **SHIFT TANGENT BY AN ORTHOGONAL NULL VECTOR**,

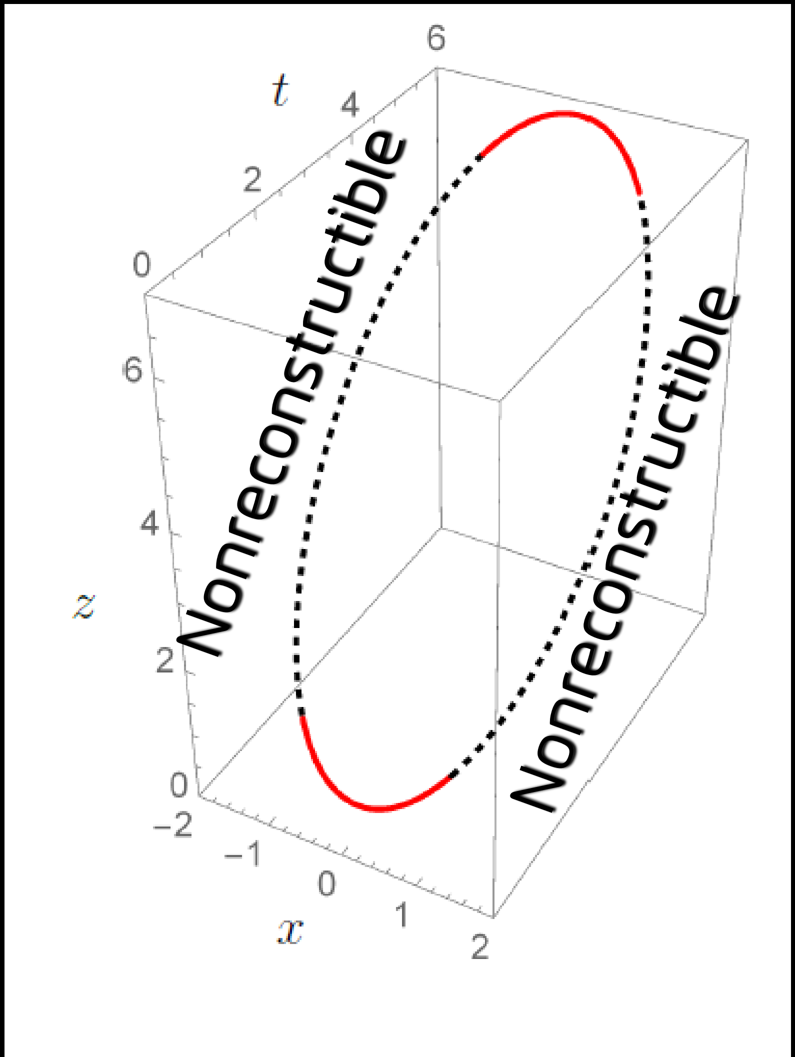
$$\boxed{u \to U \equiv u + n, \quad n \cdot n = 0, \quad n \cdot u = 0}$$

$$\Rightarrow U \cdot U = u \cdot u$$

[Headrick,Myers,Wien]

Reconstruction Achieved

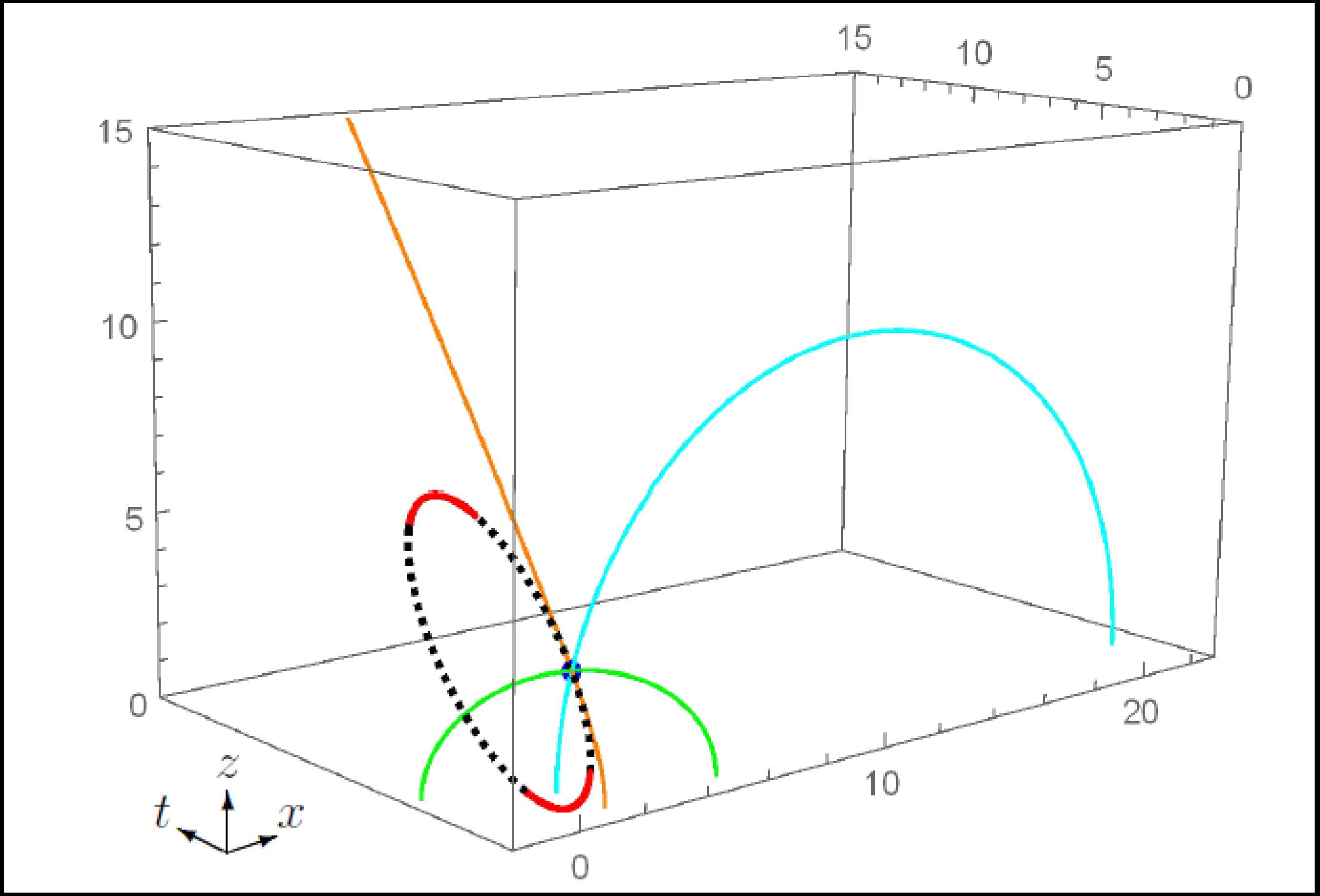
We find that on segments that are NOT reconstructible using STANDARD hole-ography, `null vector alignment' allows us to **SHOOT GEODESICS THAT DO REACH THE POINCARÉ BOUNDARY**



This can in fact be done in **INFINITELY MANY WAYS**:
at each point on the curve, there is a one-parameter family of viable geodesics, parametrized by one component of null vector n

Reconstruction Achieved

`Null vector alignment’ allows us to **SHOOT GEODESICS THAT DO REACH THE POINCARÉ BOUNDARY**:



We thus conclude that **ANY CURVE WITHIN THE POINCARÉ WEDGE CAN BE FULLY RECONS-TRUCTED**

Poincaré at Varying Time

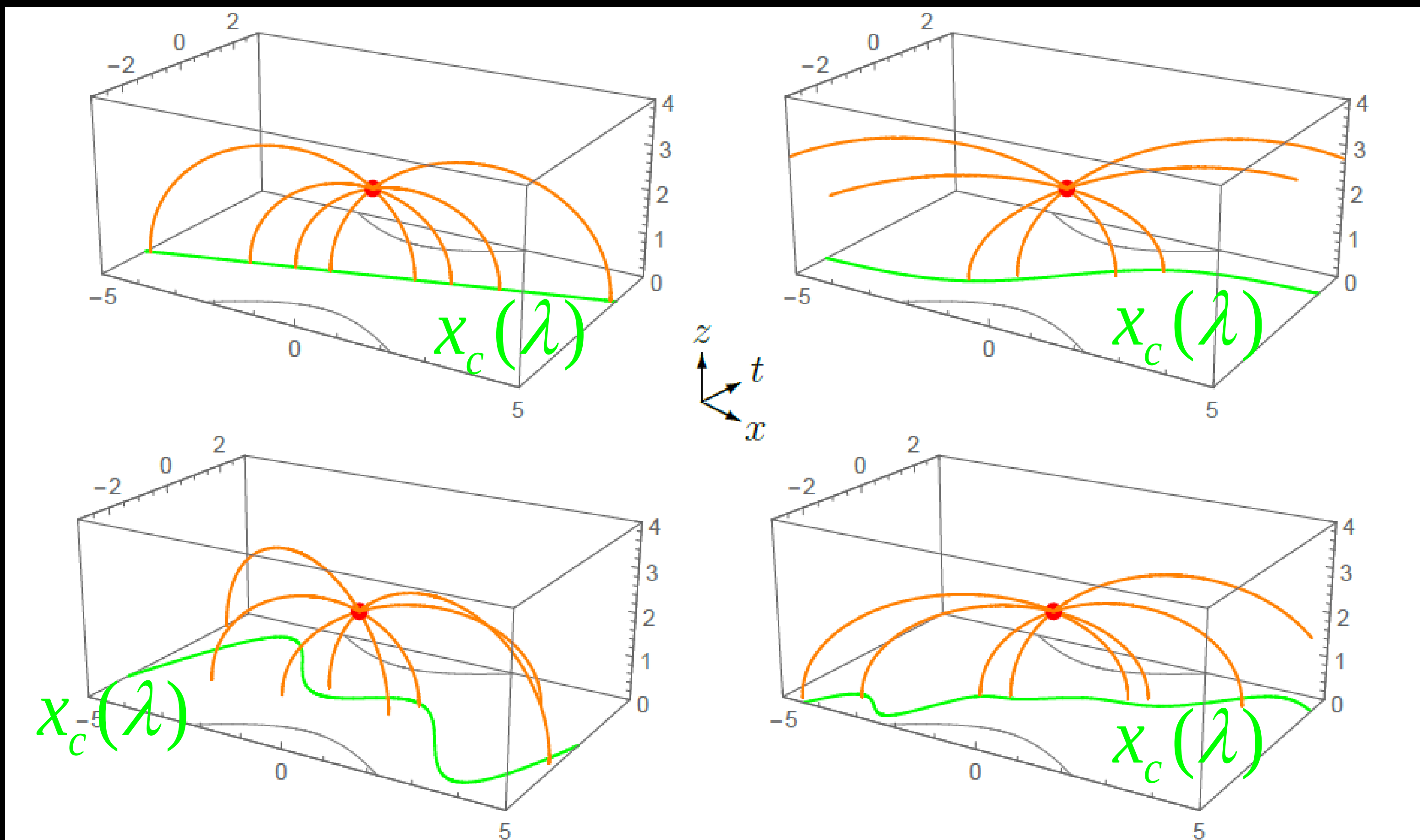
Other novelties:

1) For OPEN curves, the value of the **BOUNDARY TERM** (= total-derivative) **DEPENDS ON THE CHOICE OF NULL VECTOR** $n(\lambda)$

$$\boxed{A = E - f(\lambda_f, n(\lambda_f)) + f(\lambda_i, n(\lambda_i)) \equiv \mathcal{E}}$$

Poincaré at Varying Time

2) There are **INFINITELY MANY WAYS TO DESCRIBE ANY BULK POINT**, parametrized by choice of CENTER CURVE



Poincaré at Varying Time

3) Families of CFT intervals that correspond to BULK POINTS can be obtained by extremizing **ACTION BASED ON NORMAL CURVATURE**

$$\boxed{I = \int d\lambda \sqrt{\frac{x_c'^2(\lambda) - \ell_x'^2(\lambda) + t_c'^2(\lambda) - \ell_t'^2(\lambda)}{\ell_x'^2(\lambda) + \ell_t'^2(\lambda)}}}$$

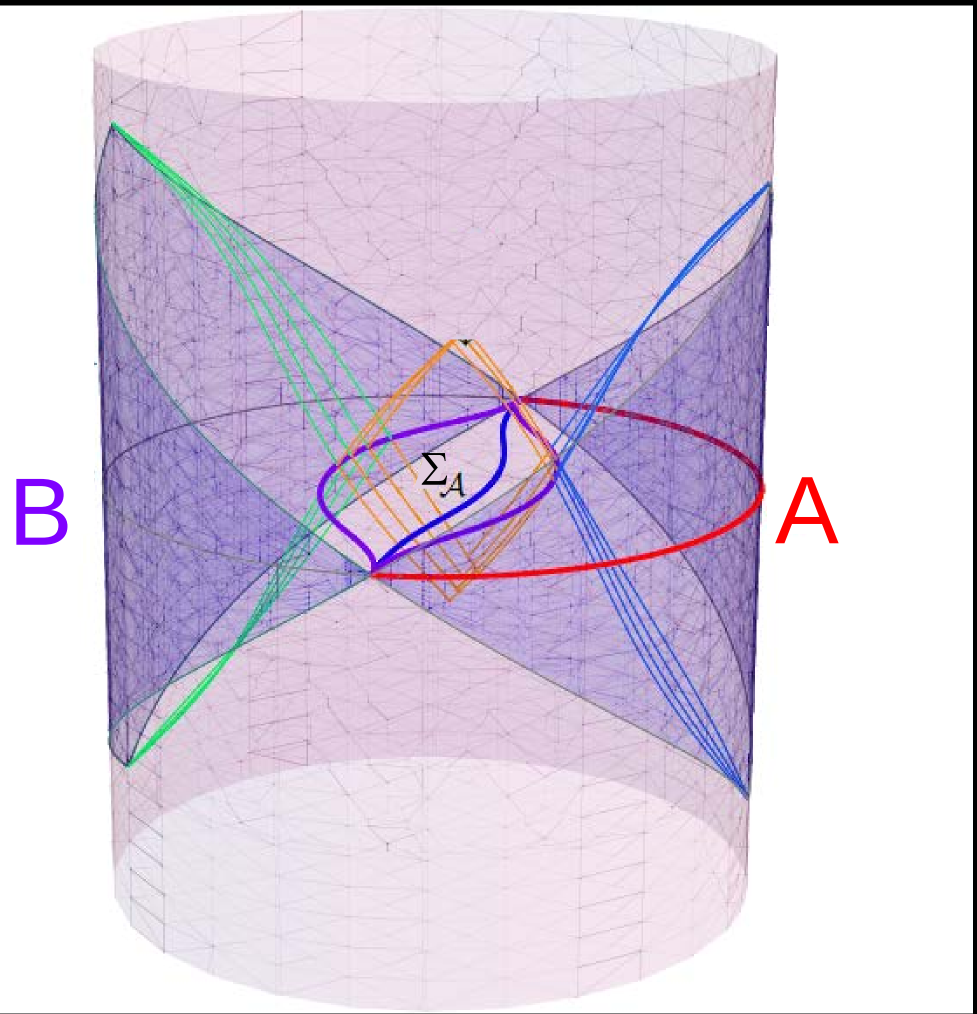
Generalizes fixed-time action of [Czech,Lamprou]

PART 2

ENTANGLEMENT WEDGE

Poincaré AdS is a particular example of an **ENTANGLEMENT WEDGE**, believed to be the **BULK region dual to a CFT region A**

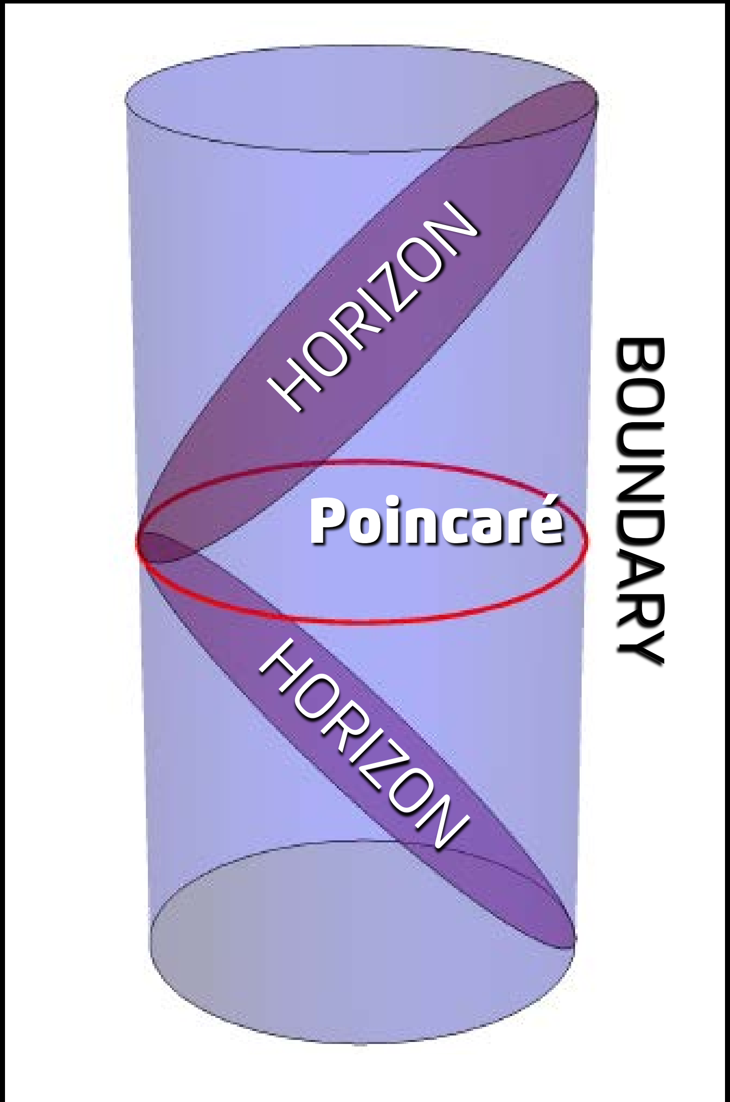
[Headrick,Hubeny,Lawrence, Rangamani; Dong,Harlow,Wall]

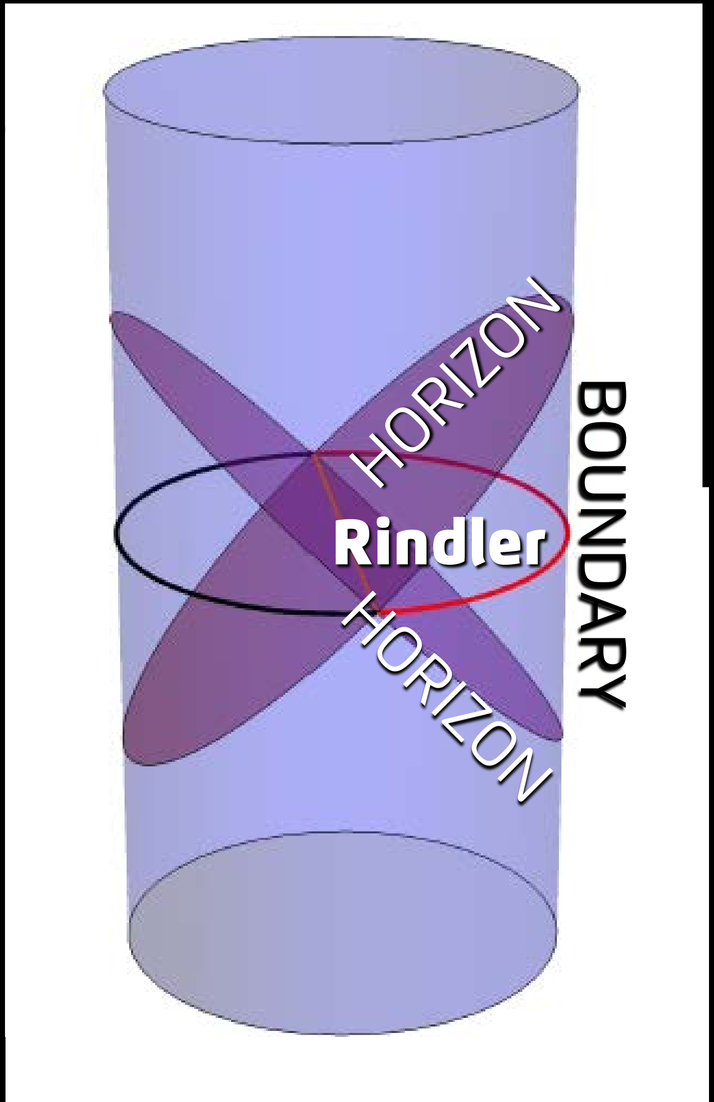


Defined as the bulk region that is connected to A and is spacelike separated from the minimal surface Σ_A associated to A

In pure AdS, taking A to be ALL of space, the **entanglement wedge IS the Poincaré wedge**

But if region A is smaller, **entanglement wedge IS smaller: RINDLER WEDGE**

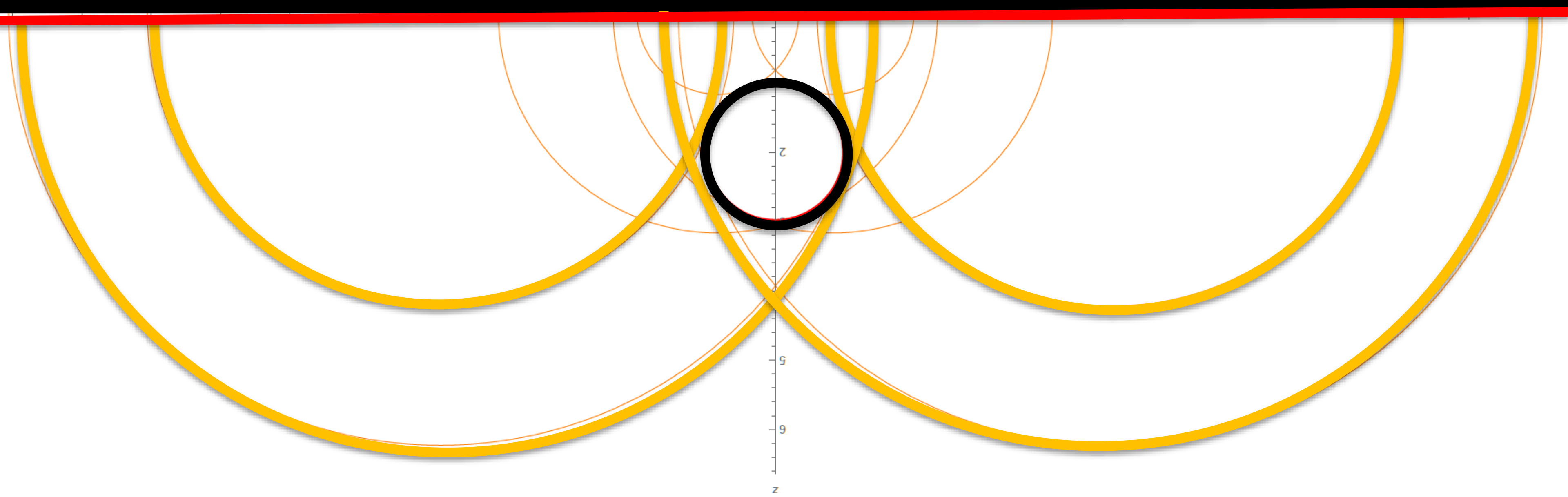




Failure of Reconstruction

Clearly the problem of **GEODESICS EXITING THE WEDGE** will be **A LOT WORSE**, even at constant time!

Compare Poincaré:



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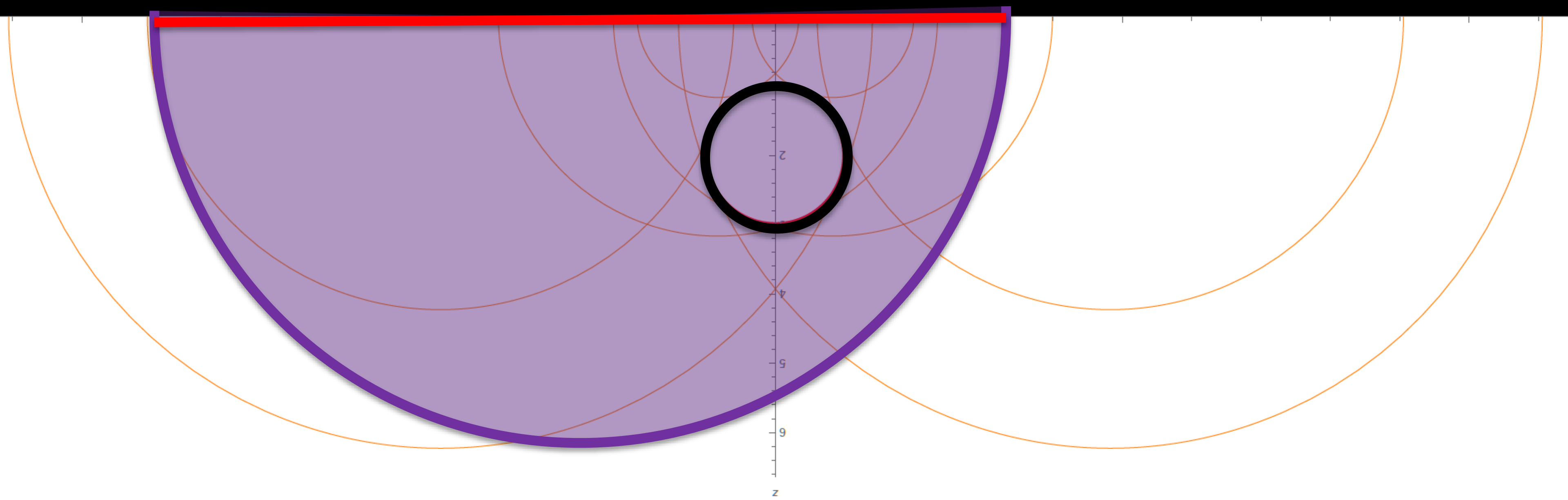
Compare Poincaré against Rindler:



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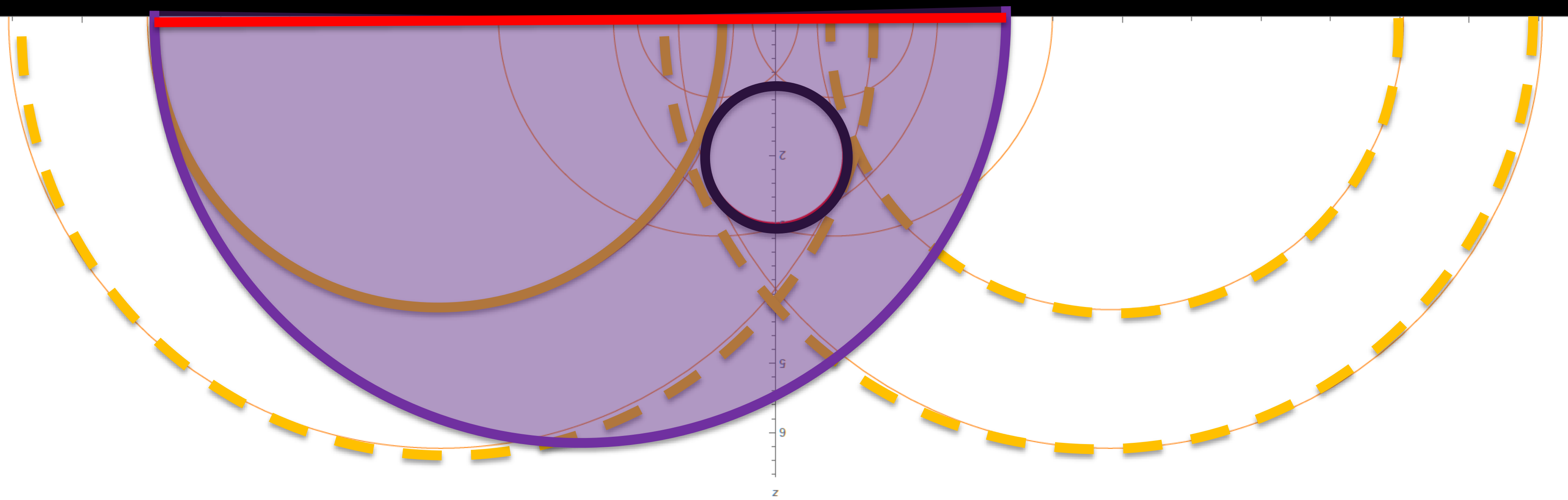


Failure of Reconstruction

Clearly the problem of **GEODESICS EXITING THE WEDGE** will be **A LOT WORSE**, even at constant time!

In coordinates adapted the the **RINDLER WEDGE**, the AdS metric is

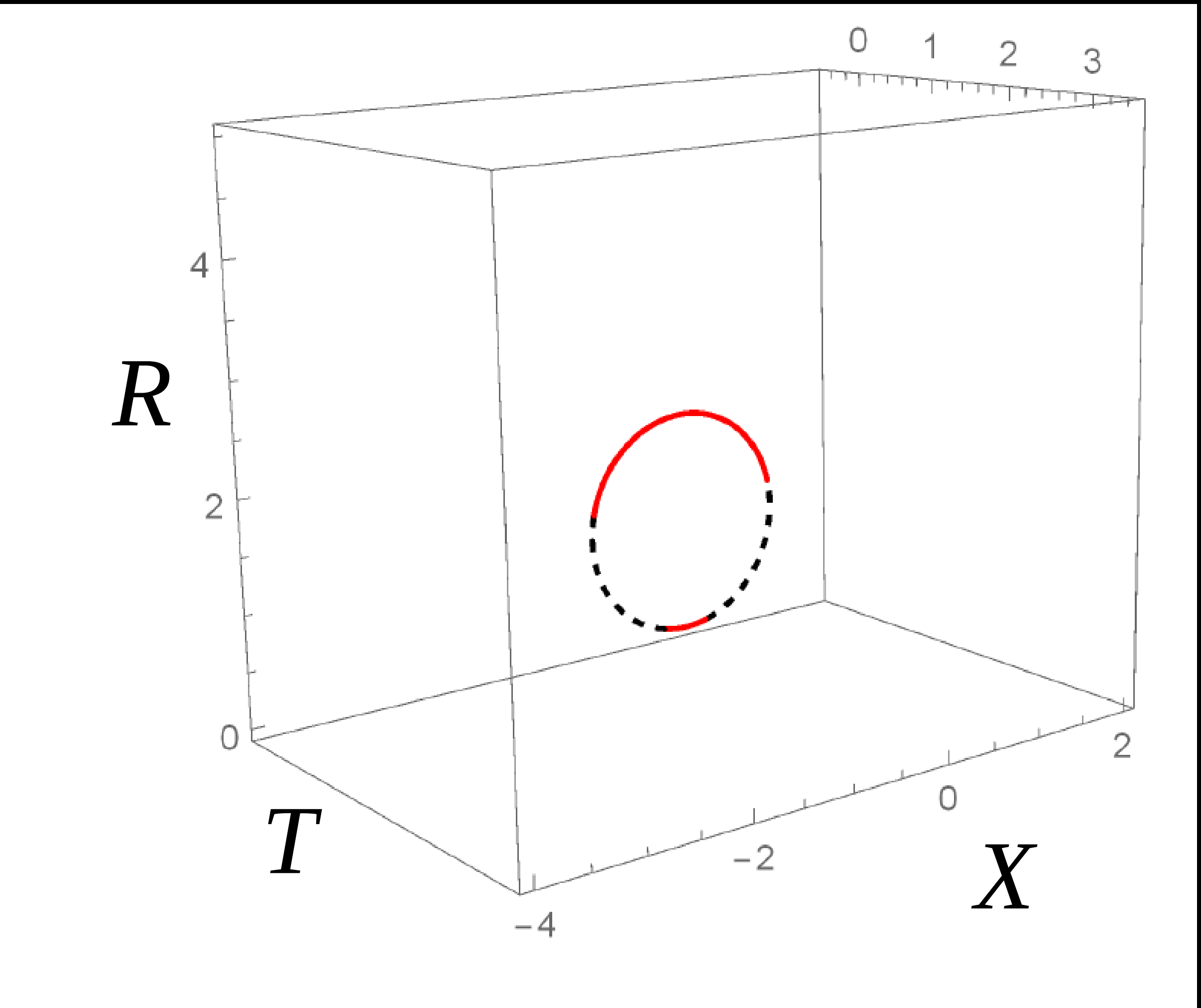
$$ds^2 = -R^2 dT^2 + \left(1 + R^2\right) dX^2 + \frac{dR^2}{1 + R^2}$$

$$-\infty < T, X < \infty, \quad 0 \le R < \infty$$

and the **CONDITION FOR RECONSTRUCTIBILITY** is

$$\boxed{R^2 \left(1 + R^2\right)^2 X'^2 > R'^2}$$

Failure of Reconstruction

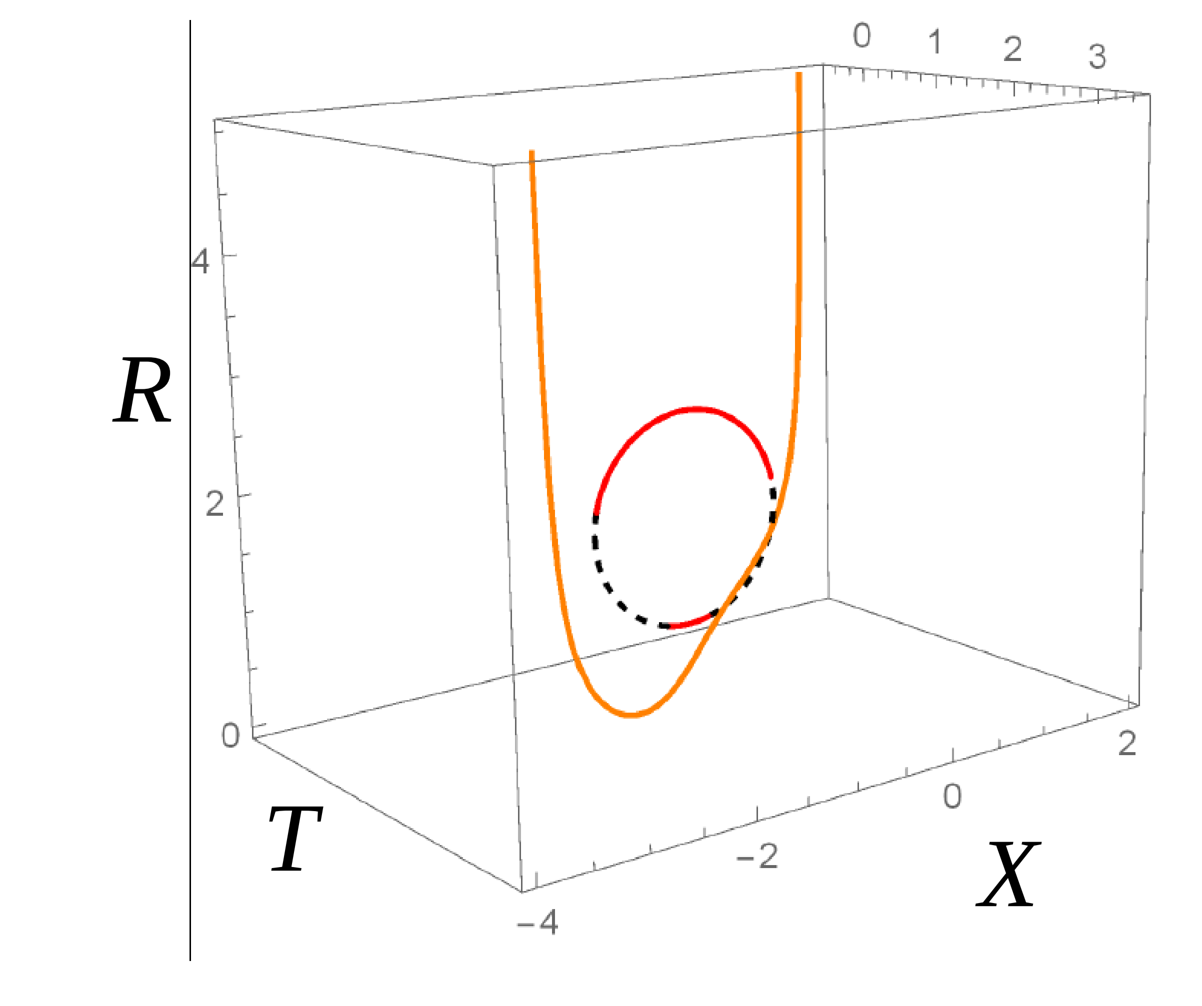


E.g., circle at constant time **CANNOT BE RECONSTRUCTED** on the sides

Reconstruction Achieved

`Null vector alignment' again **SAVES THE DAY**!!

It allows us to **SHOOT GEODESICS THAT DO REACH THE RINDLER BOUNDARY**:



(Again, this can be done in infinitely many different ways)

We thus conclude that **ANY CURVE WITHIN THE RINDLER WEDGE CAN BE FULLY RECONSTRUCTED**

CONCLUSIONS

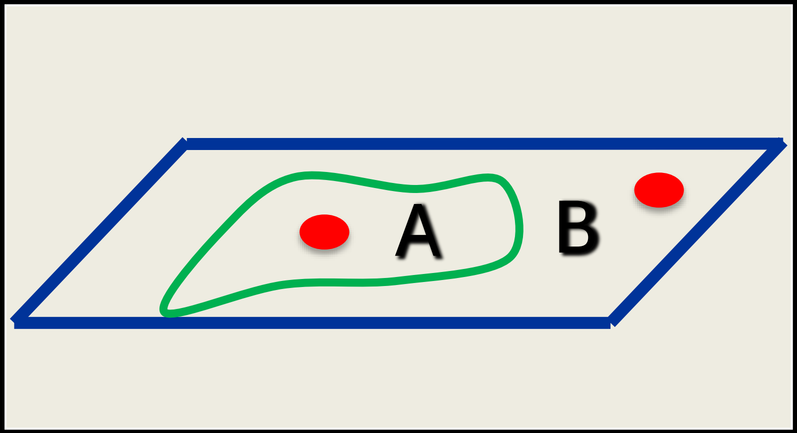
- We have extended previous analyses of **hole-ography** to **ARBITRARY SPACELIKE CURVES in Poincaré AdS**
- We discovered that **hole-ography faces a serious challenge** due to geodesics exiting the Poincaré wedge
- We showed that this **CHALLENGE CAN BE FULLY OVERCOME** by reorienting the geodesics appropriately
- SAME procedure allows **hole-ographic reconstruction** of **ARBITRARY SPACELIKE CURVES in ANY ENTANGLEMENT WEDGE**

BACKUP SLIDES

Entanglement in QFT

Normally one studies a QFT via correlators of local operators

$$\langle O_1(x_1)\cdots O_n(x_n)\rangle \equiv \langle \Omega | O_1(x_1)\cdots O_n(x_n) | \Omega \rangle$$



Usually $\langle \Omega | O(x_i) | \Omega \rangle = 0$

but

$$\langle \Omega | O(x_1) O(x_2) | \Omega \rangle \neq 0$$

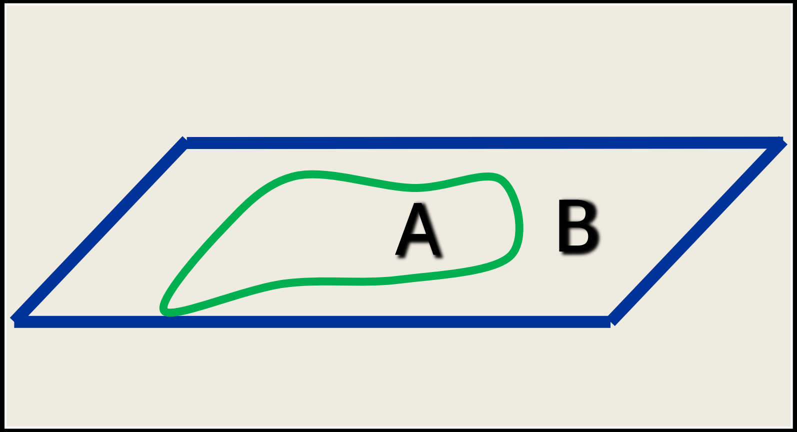
This is precisely BECAUSE the fields are **ENTANGLED**

The entanglement entropy $S(A) = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$

allows us to examine this entanglement **DIRECTLY**

Entanglement in QFT

Entanglement entropy in QFT is **UV divergent** (due to infinite number of degrees of freedom), and must be REGULARIZED



And it is **DIFFICULT TO COMPUTE**, even at weak coupling!

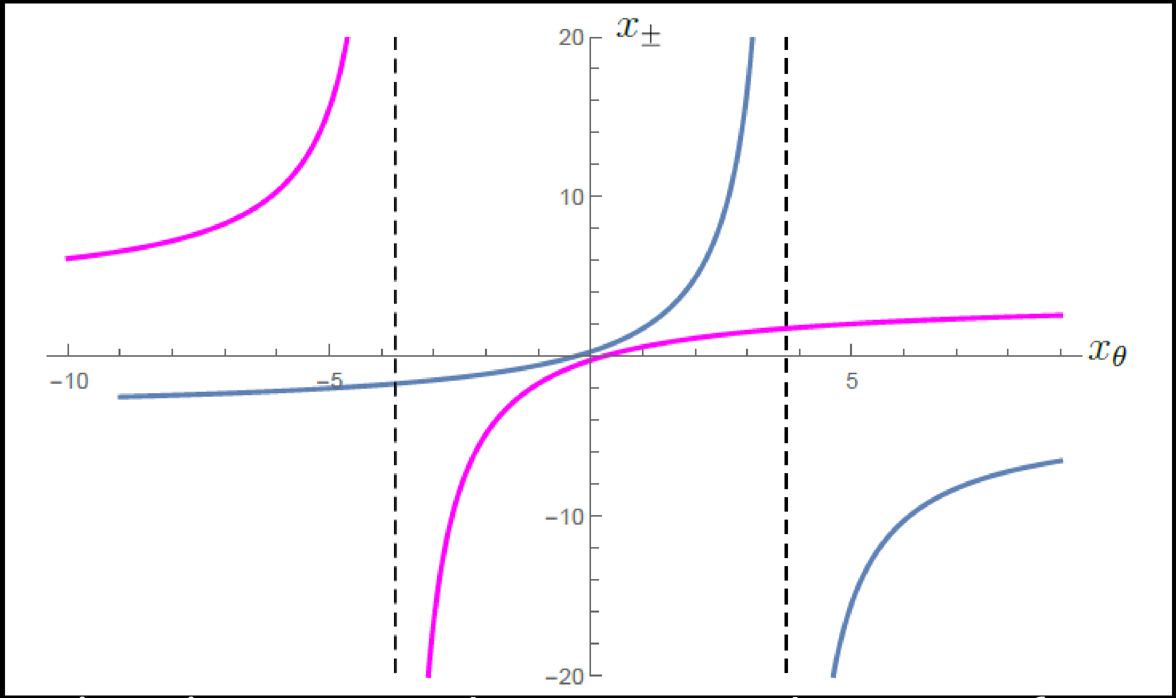
The entanglement entropy $S(A) = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$

allows us to examine this entanglement **DIRECTLY**

Poincaré at Fixed Time

We do find some novelties:

1) CLOSED curves are described by **a family of CFT intervals that covers the *x*-axis AT LEAST TWICE**



Upper portion is covered on second pass: after one of the endpoints crosses through $\pm\infty$

Poincaré at Fixed Time

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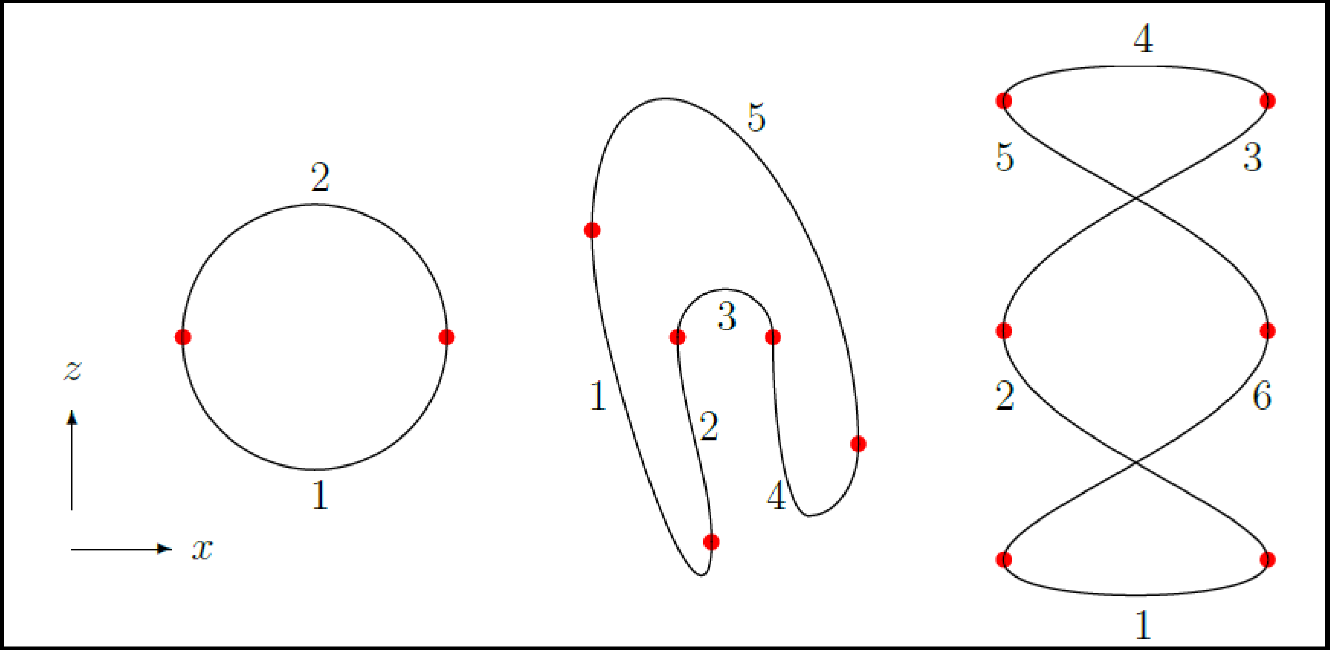
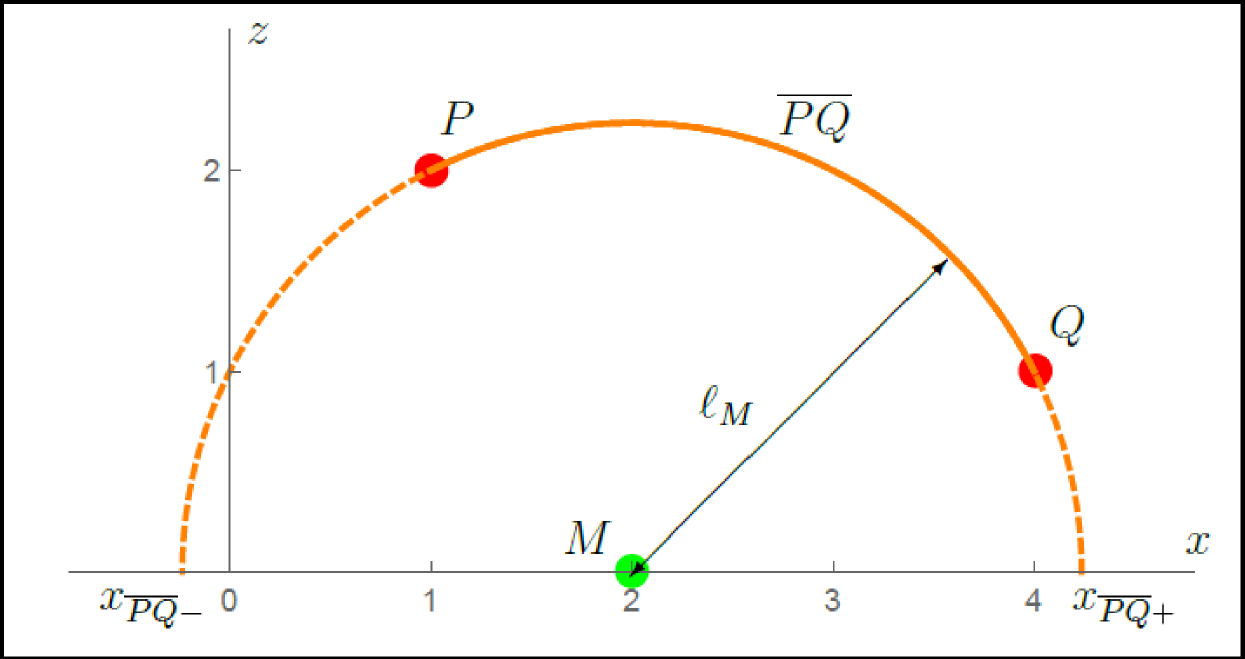


Cover the x-axis *N* times for closed curve that BECOMES VERTICAL at *N* points

Poincaré at Fixed Time

4) Can compute **DISTANCES** (in either case) using **DIFFERENTIAL ENTROPY**

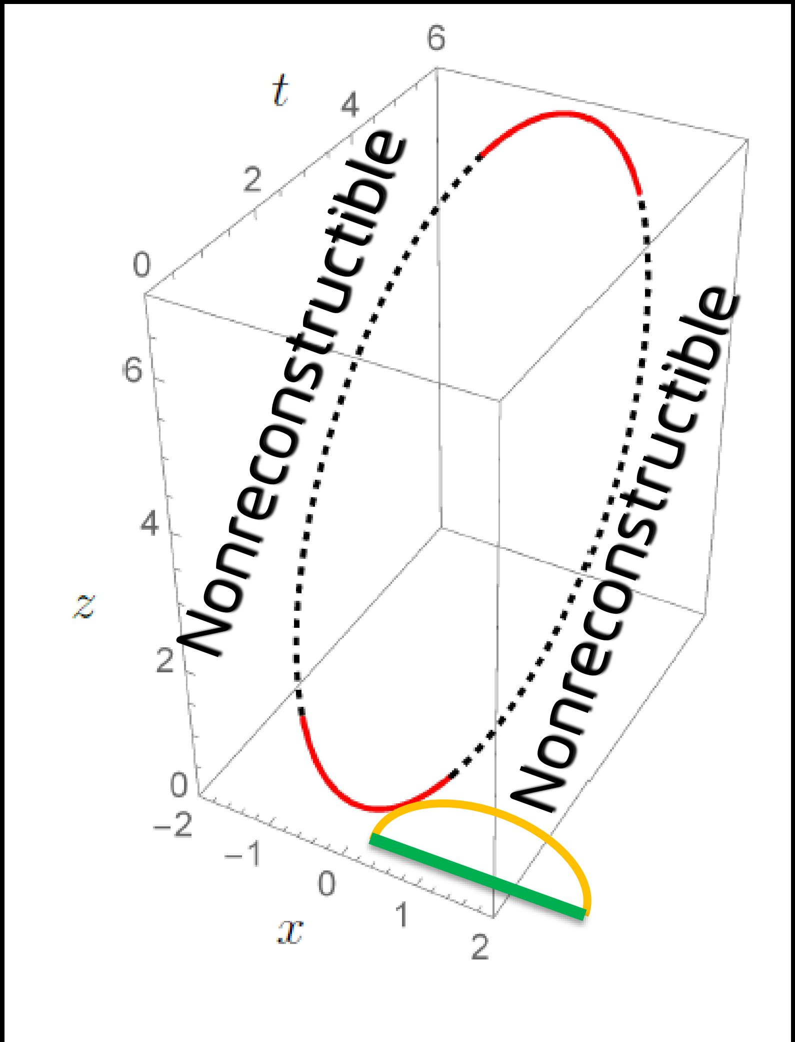


$$\boxed{d(P,Q) = \frac{1}{2} E\left[\ell_{PQ}\right]}$$

[Czech,Lamprou]

Failure of Reconstruction

Tangent vector to curve is $\left(t'(\lambda), x'(\lambda), z'(\lambda)\right)$, and is **SPACELIKE**, $-t'^2 + x'^2 + z'^2 > 0$.



- Geodesics **REACHING THE BOUNDARY** are boosted semicircles, whose PROJECTION TO BOUNDARY is a **SPACELIKE** line
- So in order for BOTH endpoints of the geodesic tangent to curve to REACH BOUNDARY, its BOUNDARY PROJECTION **must also be SPACELIKE**

$$-t'^2 + x'^2 > 0$$