UV DIVERGENCES IN SUBREGION COMPLEXITY

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20 years of AdS/CFT !

hep-th/9711200

Holographic duality

[Maldacena; Witten; Gubser,Klebanov, Polyakov; Susskind; 't Hooft]

certain examples of Quantum Gravity	=	Quantum Field Theory

AdS/CFT

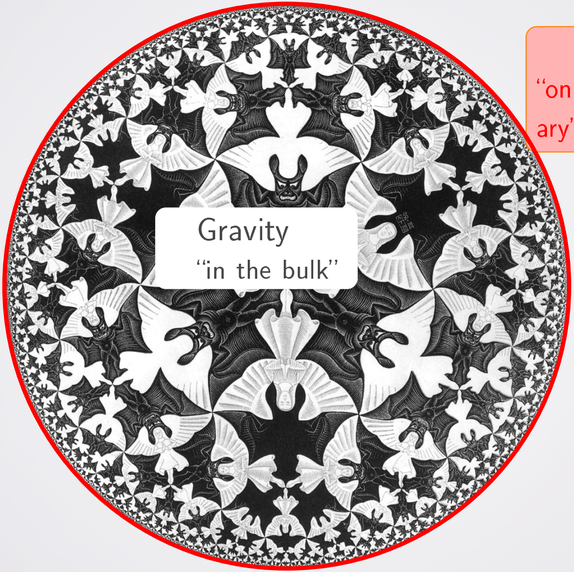
[Maldacena; Witten; Gubser,Klebanov,Polyakov; Susskind; 'tHooft]

[Escher, 1960, CLIV, "Angels and Devils"]

AdS/CFT

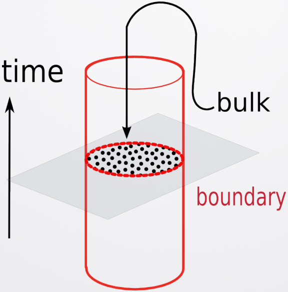
CFT
"on the boundary"

AdS/CFT



Gravity
"in the bulk"

CFT
"on the boundary"


time
bulk
boundary

- Many years of using AdS/CFT as a powerful tool understand stongly coupled CFT's
  - quark gluon plasma
  - condensed matter systems

AdS/CFT geometrizes field theoy concepts more rececently:

reconstruct bulk from boundary data?

more rececently:

reconstruct bulk from boundary data?

fundamental ingredient: entanglement

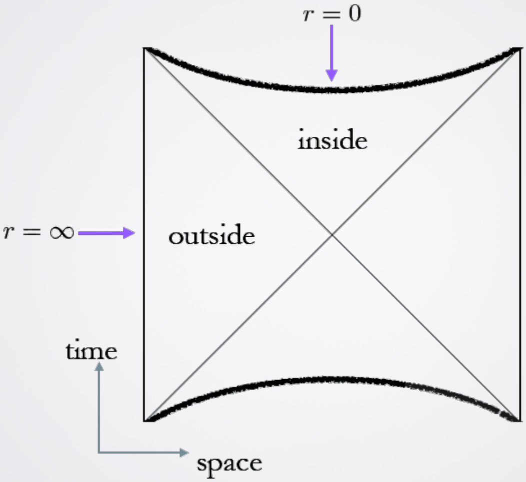
Holographic Entanglement Entropy

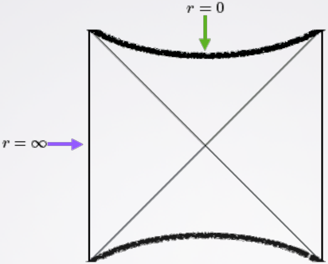
but......
What is the dual of the infinitely growing wormhole?

Entanglement is not enough !

Susskind 2014

infinitely growing wormhole

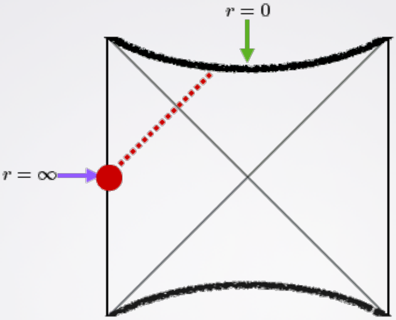




$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega_{D-2}^2$$

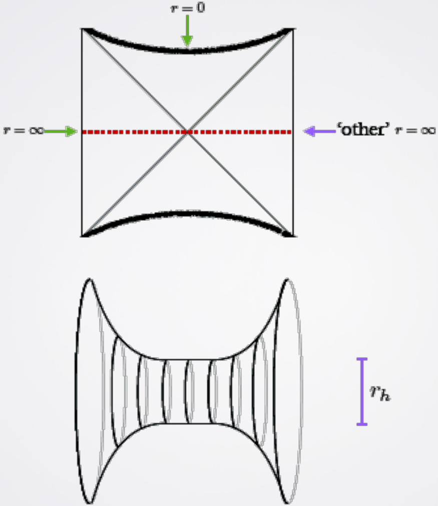
For $D = 3$

$$f(r) = \frac{1}{L^2}(r^2 - M)$$

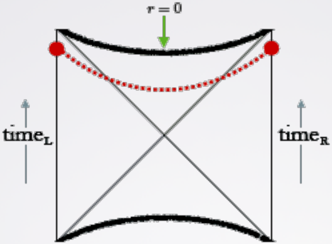


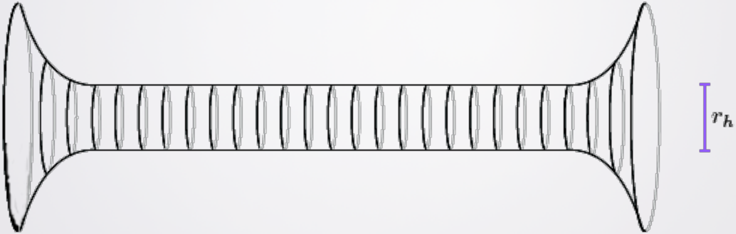
S^{D-2}

radius ∞ 0



$r = 0$
$r = \infty$
'other' $r = \infty$
r_h





wormhole length $\sim t_L + t_R$

Conjecture:

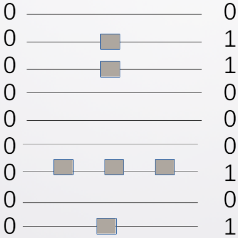
$$\text{“Size” of wormhole} = \text{complexity}$$

Complexity

- Many definitions of complexity
- here: computational complexity
- How dificult it is to prepare a state

Complexity=minimum number of gates required to reach the target state starting from some reference state

example: NOT gate



$$|\psi\rangle = U\,|\psi_0\rangle$$

unitary operator built from set of simple gates

simple reference state eg, $|00000\cdots 0\rangle$

- Universal quantum gates
- Different type of gates (Hadamard, cnot, swap, Toffoli, etc)
- Need at least 2 qubit gates

Many questions in the CFT side, active research

- does the answer depend on the type of gate?
- complexity in field theory?
- ambiguities in gravity and boundary ? connections?
- is there a renormalized holographic complexity? what is it good for?
- .....

Gravity side

Evidence for complexity= “size" of wormhole

- both grow linearly
- both can be exponentially large
- unlike entanglement entropy complexity keeps on growing after thermalization so does wormhole size

$$\frac{d\mathcal{C}}{dt} \sim M$$

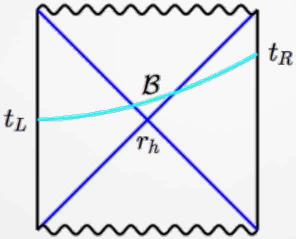
- perturb with boundary operator, growth of wormhole matches growth of complexity

Holographic complexity

how do we measure the “size" of a wormhole?

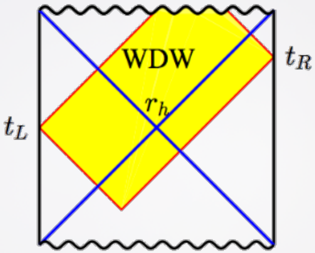
Two possible definitions

- complexity equals volume (CV) evaluate proper volume of extremal codim-one surface connecting t_L and t_R at the boundaries



$$\mathcal{C} = \frac{\mathcal{V}}{G_N \ell}$$

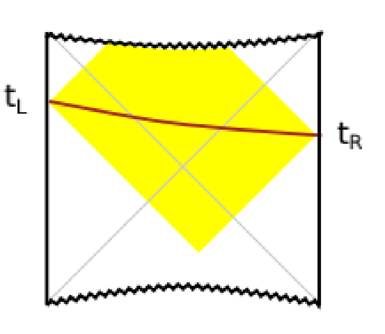
- complexity equals action (CA) evaluate gravitational action on Wheeler-DeWitt patch (domain of dependence of bulk time slice that connects t_L and t_R



$$\mathcal{C} = \frac{I_{WdW}}{\pi\hbar}$$

★ both of these prescriptions probe black hole interior

Currently CA is favored —it does not need any additional scale to be introduced by hand.



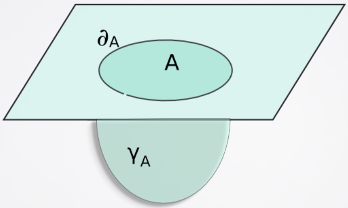
Subregion complexity
Myers et. al 2016

- Complexity of a mixed state obtained by considering a subregion of a pure state (similar to entanglement entropy)
- Natural geometric quantity on the bulk side.

Field theory meaning is under research

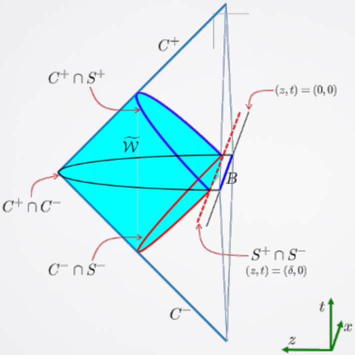
- Fidelity susceptibility.
- Minimal purification of density matrix.
- cMERA.

- Subregion CV :
  Complexity = max volume enclosed by HRT surface.



$$\mathcal{C}_V(A) = \frac{V(\gamma_A)}{G_N \ell}$$

- Subregion CA:
  Complexity = Action of the intersection between entanglement wedge and the WdW patch.



$$\mathcal{C}_A(A) = \frac{I_{WDW}(\widetilde{\mathcal{W}})}{\pi\hbar}$$

UV divergences in subregion complexity

Why do we care?

In holographic entanglement entropy:

- The area law is found at leading order UV divergence
- The structure of UV divergence is tractable and contains important information

$$S_A = c_0 \frac{L^{d-2}}{\delta^{d-2}} + c_1 \frac{L^{d-4}}{\delta^{d-4}} + \cdots + \begin{cases} \tilde{S}_A \log \frac{\ell}{\epsilon} + \ldots \text{d even} \\ \tilde{S}_A + \ldots \text{d odd} \end{cases}$$

$\Downarrow$ cutoff dep. coeff.

$\Downarrow$ universal coeff.

- Additional universal logarithmic UV divergence when boundary region has sharp corners:

$$S_{\text{cone}} = a(\Omega) \log \frac{\delta}{L}$$

$a(\Omega)$ satisfies

$$\begin{aligned}
&\lim_{\Omega \to 0} a(\Omega) = \frac{\kappa}{\Omega} + \dots \\
&a(\Omega) = a(2\pi - \Omega) \quad \text{since for pure states } S_A = S_{\bar{A}} \\
&a(\pi - \Omega) \sim \sigma(\pi - \Omega)^2
\end{aligned}$$

Two interesting facts:

- ratio κ/σ is the same in all holographic models studied
- ratio $\frac{\sigma}{C_T} = \frac{\pi^2}{24}$ is universal constant for $D = 3$ CFT's

Universal UV divergences capture information about the CFT

UV Divergence for Holographic Complexity

Motivation:

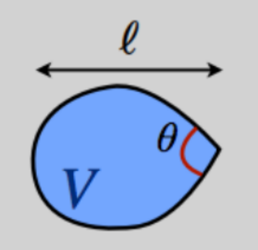
- We expect that a detailed study of the UV divergences of holographic complexitywill also reveal significant universal quantities

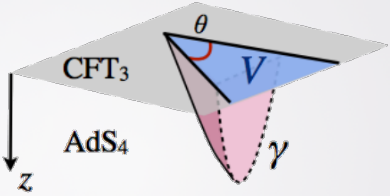
- This involves studying UV divergences in holographic CFT's in diverse dimensions, with higher derivatives, etc

- The present work is just a first step in this program

Corner Contributions: 3D Kink

Kink: $\rho \in [0, +\infty)$, $\theta \in [-\Omega, \Omega]$.





Entangling surface γ: $z = z(\rho, \theta)$.

$$\mathcal{A} = \frac{4\pi L^2}{\ell_p^2} \int_{\tilde{\delta}}^{R} \mathrm{d}\rho \int_0^{\Omega-\epsilon} \mathrm{d}\theta \frac{\sqrt{\rho^2 + \rho^2 z'^2 + \dot{z}^2}}{z^2}$$

Minimize $\mathcal{A}$ to determine γ

SUBREGION CV

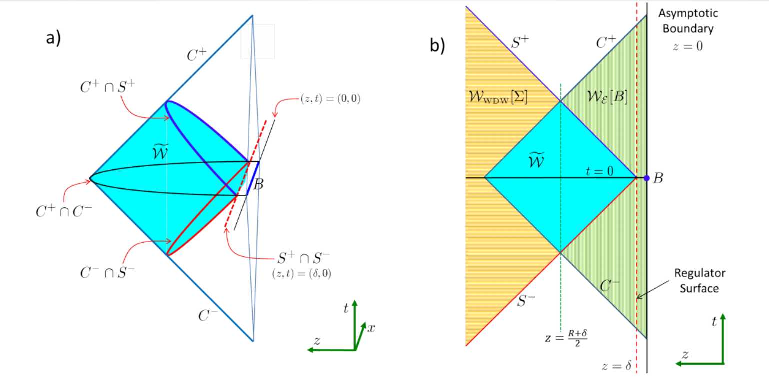
proceeds very similarly to entanglement entropy
determine RT sfce γ then calculate $V(\gamma)$
"CV" result `(Bakhshaei, Mollabashi, Shirzad)`

$$\mathcal{C}_V^{\text{kink}} = \frac{\Omega}{2}\frac{R^2}{\delta^2} + \alpha \log \frac{\delta}{R} \tag{1}$$

- For small Ω, $\alpha \sim \frac{2.188}{\Omega}$.
- $\mathcal{C}_V(A) + \mathcal{C}_V(\bar{A}) = \mathcal{C}_V(\Sigma) \to \alpha(\Omega) = -\alpha(\pi - \Omega)$
- For near smooth limit $\Omega \to \pi/2$, $\alpha(\Omega) = \epsilon + \mathcal{O}(\epsilon^3)$

subregion CA

- extremly technical calculation
- intersection of WdW patch and EE wedge
- careful evaluation of action : null boundaries



"CA" result (Cáceres, Xiao)

$$
\begin{aligned}
\mathcal{C}_A^{\text{kink}} = \\
&A\frac{R^2}{\delta^2} + B\frac{R}{\delta} + C\log\frac{\delta}{R} + D\log^2\frac{\delta}{R} \\
&+ \log\frac{\alpha\delta}{L}\left[A'\frac{R^2}{\delta^2} + B'\frac{R}{\delta} + C'\log\frac{\delta}{R} + C_\alpha\right] \\
&+ \log\frac{\beta\delta}{L}\left[C''\log\frac{\delta}{R} + C_\beta\right].
\end{aligned}
$$

Comments:

- A is the volume contribution, same as that in "CV".
- $B = B^{\text{kink}} + b$, where b is a constant, representing lowest order boundary contribution. B^{kink} is a corner contribution – dies away in the near smooth limit .
- "Universal" terms are $\{C, C_\alpha, C_\beta, D\}$.
- $C + C_\alpha + C_\beta = C^{\text{kink}} + c$
  c is a finite piece left in the near smooth limit, implying a boundary contribution.
  C^{kink} is a corner contribution.

Summary of the two approaches:

- $\mathcal{C}_V(A) + \mathcal{C}_V(\bar{A}) = \mathcal{C}_V(\Sigma)$. No such constraint for $\mathcal{C}_A$.
- $\frac{R}{\delta}$ divergence in "CA" , absent in "CV" .
- Cutoff dependent corner contribution to $\log \frac{R}{\delta}$ in "CA" .
- Additional divergence structure in "CA", i.e. $\log^2 \frac{\delta}{R}$.

Left to do:

- study possible geometric or field theoretic interpretation of coefficients

Would like to understand:

- issue with normalization of null vectors

Future Work

1. Finite region with corners (football shape).
   universal contributions?.
2. Higher dimensions and other geometries.
   Check the spacetime dimension and curvature dependence of those coefficients.
3. Higher dimensional corners (trihedral corners and so on).
   Singular, and MORE singular!
4. Field theory understanding (cMERA).
   A brand new subject in field theory, relating to quantum computing.