Non abelian T-dualities in Gauged Linear Sigma Models

Nana Cabo Bizet

División de Ciencias e Ingenierías,
Universidad de Guanajuato.

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Collaborators: Aldo Martínez Merino, Leopoldo Pando Zayas and Roberto Santos Silva.

Motivation

$\boxed{\text{T-Duality}}$ is a $R \to 1/R$ symmetry of string theory which exchanges winding modes with momentum modes.

$\boxed{\text{Mirror symmetry:}}$ Exchanges Kähler deformations with complex deformations on dual manifolds. It is connected to T-duality [Morrison, Plesser, 95][Strominger, Yau, Zaslow, 96].

The $\boxed{\text{Gauged Linear Sigma Models}}$(GLSMs) [Witten, 93] in 2D with $\mathcal{N} = 2$ are a powerful tool to describe strings propagating in a Kähler manifold. Under the renormalization group GLSMs have IR points with conformal invariance.

Motivation

[Hori, Vafa, 00] developed a proof of mirror symmetry for supersymmetric sigma models on Kähler manifolds, based on studying abelian T-dual models of the GLSMs.

We study $\boxed{\text{non-abelian T-duality}}$ [Buscher, 88][de la Ossa, Quevedo, 93][Giveon, Rocek, 94] in GLSMs.

We aim to explore the connections between GLSM non abelian T-dualities and symmetries between target manifolds of string theory (f.e. $\boxed{\text{mirror symmetry on determinantal CY varieties}}$).

Resume

We describe the abelian T-duality on a 2D $\mathcal{N} = 2$ U(1) GLSM as a gauging of a global U(1) symmetry and the addition of lagrange multipliers (review with different treatment than in [Hori, Vafa, 00]).

We describe the non-abelian T-duality on the 2D $\mathcal{N} = 2$ U(1) GLSM by gauging global non-abelian symmetries.

The e.o.m. to describe the dual model are obtained, we solve them for the SU(2) group for a choice of vector superfield obtaining the dual model with a generated twisted superpotential, and the dual vacuum manifold of the GLSM.

Non perturbative: Making an ansatz for the instanton corrections, the effective potential for the U(1) gauge field on the original theory matches the one of the dual theory.

$\mathcal{N} = 2$ Gauged Linear Sigma Models in 2d

The reduction to 2d of $\mathcal{N} = 1$ 4d supersymmetric $U(1)$ gauge field theory gives rise to a $\mathcal{N} = 2$ (2,2) 2d theory.

The $\mathcal{N} = 1$ superfield language is employed to describe chiral superfields (csf), antichiral superfields (acsf) and vector superfields (vsf).

In the duality process twisted chiral superfields (twisted-csf) arise.

The csf satisfies $\bar{D}_{\dot{\alpha}}\Phi = 0$ and the acsf $D_{\dot{\alpha}}\bar{\Phi} = 0$.

The vsf $V^{\dagger} = V$.

The twisted-csf satisfy $D_{+}X = \bar{D}_{-}X = 0$ and the twisted-acsf satisfy $\bar{D}_{+}\bar{X} = D_{-}\bar{X} = 0$.

$\mathcal{N} = 2$ Gauged linear sigma models in 2d

The Lagrangian of a GLSM with gauge group $U(1)$ with vsf V_0 and N csfs Φ_i with charge Q_i can be written as [Hori, Vafa, 00]

$$L_0 = \int d^4\theta \left(\sum_{i=1}^{N} \bar{\Phi}_i e^{2Q_i V_0} \Phi_i - \frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 \right) - \frac{1}{2} \int d^2\tilde{\theta} t \Sigma_0 + c.c., \quad (1)$$

where $t = r - i\theta$. The parameters are the $U(1)$ gauge coupling e, the Fayet-Iliopoulos(FI) term r and the Theta angle θ. The twisted field strength for V_0 is given by $\Sigma_0 = \frac{1}{2}\bar{D}_+ D_- V_0$.

The classical theory is invariant under vector $U(1)_V$ and axial vector $U(1)_A$ R-symmetries.

There are $N-1$ global $U(1)$ symmetries. For particular charges Q_i these symmetries could be bigger.

Abelian T-duality in the GLSM

Exercise: Implement T-Duality of a $(2,2)$ GLSM with gauge group $U(1)$ with vsf V_0 and 2 csfs $\Phi_{1,2}$ with charges Q_1 and Q_2.

There is $\boxed{\text{1 global } U(1)}$: the phase rotations of Φ_1 and Φ_2 modulo the $U(1)$ gauge transformations.

We gauge the remnant global symmetry, obtaining the vector field V and adding a Lagrange multiplier.

Using the gauge freedom we fix the phase transformation of Φ_2, s.t. Φ_1 is the charged field under the global $U(1)$. The T-dualization is implemented with respect to Φ_1 as

$$\begin{aligned} L_1 &= \int d^4\theta \left(\bar{\Phi}_1 e^{2Q_0 V_0 + 2QV} \Phi_1 + \bar{\Phi}_2 e^{2Q_{0,2} V_0} \Phi_2 + \Psi\Sigma + \bar{\Psi}\bar{\Sigma} - \frac{1}{2e^2}\bar{\Sigma}_0 \Sigma_0 \right) \quad (2) \\ &+ \frac{1}{2}\left(- \int d^2\tilde{\theta}\, t \Sigma_0 + c.c. \right). \end{aligned}$$

Abelian T-duality in the GLSM

Integrating the lagrange multipliers Ψ and $\bar{\Psi}$ one obtains the original model.

Dual model: The equation of motion obtained by integrating out V is

$$\bar{\Phi}_1 e^{2Q_0 V_0 + 2QV} \Phi_1 = \frac{\Lambda + \bar{\Lambda}}{2Q},$$

with $\Lambda = \bar{D}_+ D_- \Psi$ and $\bar{\Lambda} = \bar{D}_- D_+ \bar{\Psi}$. The vsf can be written as

$$V = \frac{1}{2Q} \ln \frac{\Lambda + \bar{\Lambda}}{2Q} - \frac{1}{2Q} \ln \bar{\Phi} e^{2Q_0 V_0} \Phi. \tag{3}$$

Abelian T-duality in the GLSM

The dual lagrangian can be obtained as

$$
\begin{aligned}
L_1 &= \int d^4\theta \left(-\frac{\Lambda+\bar{\Lambda}}{2Q} \ln\left(\frac{\Lambda+\bar{\Lambda}}{2Q}\right) + \bar{\Phi}_2 e^{2Q_{0,2}V_0}\Phi_2 \right) \\
&+ \frac{1}{2}\left(\int d^2\tilde{\theta}(\Lambda Q_0/Q - t)\Sigma_0 + \int d^2\bar{\tilde{\theta}}(\bar{\Lambda} Q_0/Q - \bar{t})\bar{\Sigma}_0 \right), \\
&- \int d^4\theta \left(\frac{1}{2e^2}\bar{\Sigma}_0\Sigma_0 \right).
\end{aligned} \tag{4}
$$

Mirror symmetry: The csf Φ_1 is exchanged by the twisted csf Λ [Hori, Vafa, 00].

Non abelian T-duality

The action can have $\boxed{\text{non-abelian global symmetries}}$. We gauge these symmetries to describe non abelian T-dual models. $\boxed{\text{Local non-abelian transformations}}$ for superfields will be required. Consider a non abelian group with generators T_A, a chiral superfield Φ tranforms as

$$\Phi' = e^{i\Lambda}\Phi,\ \Lambda = \Lambda^A T_A,\ \bar{D}_{\dot{\alpha}}\Lambda = 0, \tag{5}$$

with Λ a csf in the adjoint of the group. As well $\bar{\Phi}' = \bar{\Phi}e^{-i\bar{\Lambda}},\ \bar{\Lambda} = \bar{\Lambda}^A T_A,\ D_\alpha\bar{\Lambda} = 0.$
A vsf transforms as

$$e^{V'} = e^{i\bar{\Lambda}}e^{V}e^{-i\Lambda}, \quad V = V^A T_A. \tag{6}$$

The twisted chiral gauge field strength $\Sigma = \frac{1}{2}\{\bar{\mathcal{D}}_+, \mathcal{D}_-\}$ and its conjugate transform as

$$\Sigma \to e^{i\Lambda}\Sigma e^{-i\Lambda},\ \bar{\Sigma} \to e^{i\bar{\Lambda}}\bar{\Sigma}e^{-i\bar{\Lambda}}. \tag{7}$$

Consider a GLSM with abelian gauge group $U(1)$, N chiral superfields $\Phi_{k,i}$ with charges Q_k, $\sum_k n_k = N$, $i = 1, ..., n_k$ where n_k is the number of superfields with charge n_k.

There is a non-abelian global symmetry G that can be gauged to obtain the vsf V. By adding a Lagrange multiplier Ψ one gets a new action.

$$\begin{aligned} L_2 &= \int d^4\theta \left(\sum_k \bar{\Phi}_{k,i} (e^{2Q_k V_0 + V})_{ij} \Phi_{k,j} + \mathrm{tr}(\Psi\Sigma) + \mathrm{tr}(\bar{\Psi}\bar{\Sigma}) \right) . \\ &- \int d^4\theta \frac{1}{2e^2} \bar{\Sigma}_0 \Sigma_0 + \frac{1}{2} \left(- \int d^2\tilde{\theta} t \Sigma_0 + c.c. \right) . \end{aligned} \tag{8}$$

Integrating out Ψ the original action is obtained.

Non abelian T-duality

Integrating out V the dual action is obtained.

For groups with traceless generators T_a integrating V one gets

$$0 = D_+ \bar{D}_- \bar{\Psi}_a \Delta V_a^\dagger + D_- \bar{D}_+ \Psi_a \Delta V_a + (\bar{\Phi} e^{2QV_0} e^V T_a \Phi) \Delta V_a. \quad (9)$$

The variation $\Delta V = e^{-V} \delta e^V$ is a gauge invariant vsf variation. Under the condition $\Delta V_a = (\Delta V_a)^\dagger$ (9) is simplified.

Let us work now for the SU(2) global symmetry case: This requires two chiral sf with equal charge under the U(1) gauge symmetry.

GLSM with and SU(2) symmetry

We consider a GLSM with csf Φ_i and lagrangian

$$\begin{aligned} L_{model} &= \int d^4\theta \left(\bar{\Phi}_i (e^{2QV_0+V})_{ij}\Phi_j + \mathrm{tr}(\Psi\Sigma) + \mathrm{tr}(\bar{\Psi}\bar{\Sigma})\right) \\ &- \int d^4\theta \frac{1}{2e^2}\bar{\Sigma}_0\Sigma_0 - \frac{1}{2}\left(\int d^2\tilde{\theta} t\Sigma_0 + c.c.\right). \end{aligned} \tag{10}$$

with $i,j = 1,2$, and the chiral and anti-chiral superfields (Φ_1, Φ_2) and $(\bar{\Phi}_1, \bar{\Phi}_2)$ are $SU(2)$ doublets. Integrating out V one gets:

$$(\bar{\Phi} e^{2QV_0} e^V \sigma_a \Phi) = \bar{D}_- D_+ \bar{\Psi}_a + \bar{D}_+ D_- \Psi_a. \tag{11}$$

which can be solved as

$$e^{2QV_0}\bar{\Phi}_i e^V_{ij}\Phi_j = \sqrt{(X_1+\bar{X}_1)^2 + (X_2+\bar{X}_2)^2 + (X_3+\bar{X}_3)^2}. \tag{12}$$

Direction $V = V_1\sigma_1$

For simplicity let us chose a particular direction for vsf, along σ_1 direction of $SU(2)$:

$$e^V = e^{V_1\sigma_1} = \begin{pmatrix} \cosh(V_1) & \sinh(V_1) \\ \sinh(V_1) & \cosh(V_1) \end{pmatrix}, \; |V| = V_1, \; n_1 = 1. \quad (13)$$

This gives the dual lagrangian

$$\begin{aligned} L_{dual} &= \int d^4\theta \sqrt{(X_1+\bar{X}_1)^2 + (X_2+\bar{X}_2)^2 + (X_3+\bar{X}_3)^2} + \qquad (14) \\ &+ \frac{1}{2}\int d^4\theta X_1 \ln(-(X_1+\bar{X}_1) - i(X_2+\bar{X}_2)(\bar{F}-1)/(\bar{F}+1)) + c.c. \\ &+ 2Q\int d\bar{\theta}^- d\theta^+ \left(X_1 - \frac{t}{4}\right)\Sigma_0 + 2Q\int d\bar{\theta}^+ d\theta^- \left(\bar{X}_1 - \frac{\bar{t}}{4}\right)\bar{\Sigma}_0 \\ &- \int d^4\theta \frac{1}{2e^2}\bar{\Sigma}_0\Sigma_0. \end{aligned}$$

Direction $V = V_1\sigma_1$: Effective potential for Σ_0

Considering the dual model one can obtain an effective superpotential for the gauge field V_0.

Take the ansatz for the instanton contributions to be $\Delta W = 2\mu e^{-X_1}$ [Hori, Vafa,00] giving the total twisted superpotential

$$\widetilde{W} = 2QX_1\Sigma_0 + 2\mu e^{-X_1} - t\Sigma_0. \tag{15}$$

By integrating out X_1 one gets $X_1 = -\ln(\frac{Q\Sigma_0}{\mu})$ to obtain the effective superpotential for the gauge field strength Σ_0:

$$W_{eff}(\Sigma_0) = -2Q\Sigma_0 \ln\left(\frac{Q\Sigma_0}{\mu}\right) + 2Q\Sigma_0 - t\Sigma_0. \tag{16}$$

This matches the result in the original model considering one-loop effects.

Direction $V = V_1\sigma_1$: The dual model

The twisted-csf and the twisted gauge field strength have components

$$
\begin{aligned}
X_i &= x_i + \sqrt{2}\theta^+\bar{\chi}_+ + \sqrt{2}\bar{\theta}^-\chi_- + 2\theta^+\bar{\theta}^- G_i + ..., \\
\bar{X}_i &= \bar{x}_i + \sqrt{2}\chi_+\bar{\theta}^+ + \sqrt{2}\bar{\chi}_-\theta^- + 2\theta^-\bar{\theta}^+\bar{G}_i + ..., \\
\Sigma_0 &= \sigma_0 + i\sqrt{2}\theta^+\bar{\lambda}_+ - i\sqrt{2}\bar{\theta}^-\lambda_- + 2\theta^+\bar{\theta}^-(D - iF_{01}) + ..., \\
\bar{\Sigma}_0 &= \bar{\sigma}_0 - i\sqrt{2}\lambda_+\bar{\theta}^+ + i\sqrt{2}\bar{\lambda}_-\theta^- + 2\theta^-\bar{\theta}^+(\bar{D} + i\bar{F}_{01}) + ...
\end{aligned} \tag{17}
$$

Collecting all the contributions to the $\boxed{\text{dual scalar potential}}$ one gets

$$
\begin{aligned}
U &= 8QRe(\sigma_0 G_1) + 8QRe((x_1 - t/8)(D - iF_{01})) - \frac{1}{8e^2}|D - iF_{01}|^2 + K_{i\bar{j}} G_i \bar{G}_j, \\
&= 128 Q^2 e^2 |x_1 - t/8|^2.
\end{aligned}
$$

The SUSY vacuum is given by $\sigma_0 = 0$ and $x_1 = t/8$, this determines the dual manifold.

Direction $V = |V| n_a \sigma_a$

In this case $\mathrm{tr}(\Psi\Sigma) = \frac{1}{2} X_a n_a |V|$. Integrating the vector superfield we have

$$|V| = 2QV_0 + \ln 2|\Phi_1|^2 + \ln(\mathcal{K}(X_i, \bar{X}_i, n_j)). \tag{18}$$

The dual lagrangian may be written as

$$\begin{aligned} L_{dual} &= \int d^4\theta \sqrt{(X_1+\bar{X}_1)^2 + (X_2+\bar{X}_2)^2 + (X_3+\bar{X}_3)^2} + \\ &+ \frac{1}{2}\int d^4\theta X_a n_a \ln(\mathcal{K}(X_i, \bar{X}_i, n_j)) + c.c. \\ &+ 2Q\int d\bar{\theta}^- d\theta^+ \left(X_a n_a - \frac{t}{4}\right)\Sigma_0 + 2Q\int d\bar{\theta}^+ d\theta^- \left(\bar{X}_a n_a - \frac{\bar{t}}{4}\right)\bar{\Sigma}_0 \\ &- \int d^4\theta \frac{1}{2e^2}\bar{\Sigma}_0\Sigma_0. \end{aligned} \tag{19}$$

Direction $V = |V| n_a \sigma_a$

We integrate the auxiliary fields to obtain the scalar potential

$$U = 16 Q^2 |\sigma_0|^2 \mathcal{O}(x_i, \bar{x}_i, n_j) + 128 Q^2 e^2 |x_a n_a - t/8|^2. \qquad (20)$$

The object $\mathcal{O}(x_i, \bar{x}_i, n_j)$ depends on the Kähler metric.

SUSY vacua with $U = 0$ appear at $\sigma_0 = 0$ and $x_a n_a = t/8$.

The dual manifold is given by the hyperplane in $\mathbb{C}^3$.

Conclusions

We have:

- Implemented non-Abelian T-duality at the level of GLSM, obtaining e.o.m.
- Analyzed an SU(2) model and a family of abelian-duals embedded in the non-Abelian T-duality.
- Took steps toward the inclusion of nonperturbative effects: We made an ansatz that reproduces correctly the effective potential for the U(1) gauge field V_0.

To do:

- Extend the solutions to a general direction inside the gauged group.
- Explore the geometry of the dual models.

Conclusions

Future work:

- Study the connection between the non-abelian T-duality in the GLSM to Mirror Symmetry. Clue: Mirror symmetry is an equivalence in (2,2) SUSY theories exchanging chiral with twisted chiral multiplets. Under it $Q_- \leftrightarrow \bar{Q}_-$ and $\Phi \leftrightarrow \Psi$.
- Study explicit examples of dualities for CY varieties (possibly determinantal).

Gracias