Thermodynamics of anisotropic branes

Daniel Ávila

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In collaboration with Daniel Fernández, Leonardo Patiño and Diego Trancanelli

Talk plan

- Adding flavor to AdS/CFT
- Thermodynamics of the brane
- A strongly coupled anisotropic plasma
- Thermodynamics of anisotropic branes
- Conclusions

Adding flavor to AdS/CFT

- It was previously shown by Karch and Katz that flavor degrees of freedom in the boundary gauge theory can be described, at strong coupling, in terms of flavor branes embedded in the dual geometry.

- Let us consider SYM $\mathcal{N} = 4$ with gauge group $SU(N_c)$ at finite temperature T.
- The gravity dual is generated by N_c coincident D3-branes

$$ds^2 = u^2(-f(u)dt^2 + dx^2 + dy^2 + dz^2) + \frac{du^2}{u^2 f(u)} + d\Omega_5^2,$$

with

$$f(u) = 1 - \frac{u_H^4}{u^4},$$

$$T = \frac{u_H}{\pi}.$$

- We want to add matter in the fundamental representation with N_f degrees of freedom to this theory.
- In order to do this, we need to add N_f D7-branes to the D3 background. It is necessary that $N_c \gg N_f$ in order to treat the D7-branes as probe objects.

		t	x	y	z	u	θ	φ	Ω_3
N_c	D3	$\times$	$\times$	$\times$	$\times$				
N_f	D7	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$

- To better visualize how the D7-branes behaves, let us introduce new coordinates

$$(u_H \rho)^2 = u^2 + \sqrt{u^4 - u_H^4},$$

to transform the background metric to

$$\begin{aligned}\frac{du^2}{u^2 f(u)} + d\Omega_5^2 &= \frac{d\rho^2}{\rho^2} + d\Omega_5^2 \\ &= \frac{d\rho^2}{\rho^2} + d\theta^2 + \cos^2\theta d\varphi^2 + \sin^2\theta d\Omega_3^2\end{aligned}$$

- We choose to parametrize the D7-branes world volume with $\psi(\rho) = \cos\theta(\rho)$ and constant φ.
- The induced metric on the D7-brane becomes

$$ds^2 = d\rho^2 \left(\frac{1}{\rho^2} + \frac{\psi'^2}{1 - \psi^2} \right) + (1 - \psi^2) d\Omega_3^2$$

- And thus the D7-brane action reads

$$S_{D7} \propto \int \sqrt{-g_{\text{ind}}} \propto \int d\rho \left(1 - \frac{1}{\rho^8} \right) \rho^3 (1 - \psi^2) \sqrt{1 - \psi^2 + \rho^2 \psi'^2}$$

- The EOM coming from this action can be solved numerically, giving three different types of solutions: Minkowski embedding, black hole embedding and critical embedding.

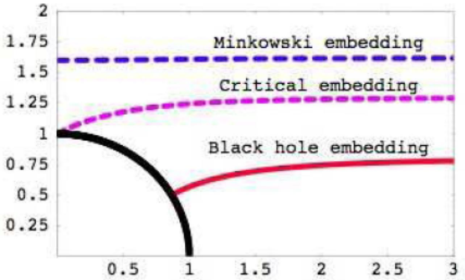
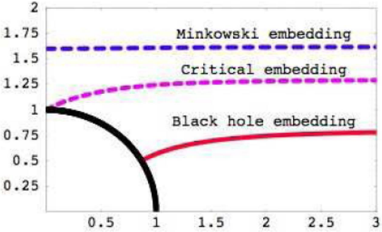


Figure : The three types of embeddings in the (R, r)-plane, where $R = \rho \cos\theta$ and $r = \rho \sin\theta$.

- The EOM for ψ is a second order differential equation, and thus we need to specify two initial conditions to obtain a unique solution. At the boundary we have

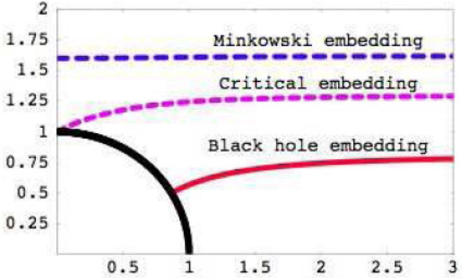
$$\psi = \frac{m}{\rho} + \frac{c}{\rho^3} + \dots$$



- It was shown by Katz and Karch that

$$m \propto M_q, \;\; c \propto \langle \bar{q} q \rangle.$$

- Given a fixed u_H, there exists a number of values for m that can give either a Minkowski or a black hole embedding, determined by the value of c.



- From the gauge theory perspective, this means that $\langle \bar{q} q \rangle$ is not a single-valued function of M_q for a given T.

Thermodynamics of the brane

- To better understand what's happening, we need to study another physical quantity.
- It is well known that the free energy of the system is related to the on-shell action by

$$F = TS_{D7}.$$

- Evaluating the action on the numerical solutions gives F as a function of T/M_q.

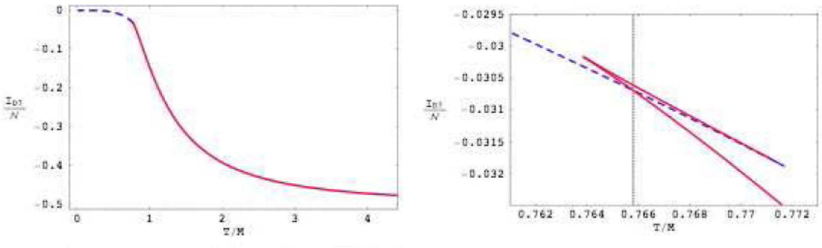


Figure : Free energy F as a funcion of T/M_q. The blue dashed line corresponds to the Minkowski embeddings and the red continuous line corresponds to the black hole embeddings. The form is typical of a first order phase transition.

- From the gauge theory perspective, what happens is that the spectrum of mesons goes from being discrete with a finite mass gap at low temperatures to a continuous spectrum of excitations at high temperatures, that is, stable mesons cease to exist.

A strongly coupled anisotropic plasma

- We consider SYM $\mathcal{N} = 4$ deformed by a theta term proportional to one of the spatial directions

$$S = S_{\mathcal{N}=4} + \frac{1}{8\pi^2} \int \theta(z) \mathrm{Tr}(F \wedge F), \;\; \theta(z) \propto z.$$

- The gravity dual is given by

$$ds^2 = \frac{e^{-\frac{1}{2}\phi}}{u^2} \left(-\mathcal{F}\mathcal{B} dt^2 + dx^2 + dy^2 + e^{-\phi} dz^2 + \frac{du^2}{\mathcal{F}} \right) + d\Omega_5^2,$$

- $\mathcal{F} = \mathcal{F}(u)$ and $\mathcal{B} = \mathcal{B}(u)$ are metric functions
- $\phi = \phi(u)$ is the SUGRA IIB dilaton field
- The axion $\chi = az$ and the 5-form $F_5 = 4(\Omega_{S^5} + \star\Omega_{S^5})$ are also turned on.
- At the boundary we have

$$\mathcal{F}(0) = \mathcal{B}(0) = 1, \qquad \phi(0) = 0,$$

and thus the background goes to $AdS_5 \times S^5$

- This geometry contains a black brane, whose horizon is found at $u = u_H$ where $\mathcal{F}$ vanishes.
- For $a = 0$, the background reduces to the black D3-brane solution

$$\mathcal{B} = 1, \qquad \phi = \chi = 0, \qquad \mathcal{F} = 1 - \frac{u^4}{u_H^4}.$$

Thermodynamics of anisotropic branes

- Now we will consider probe D7-brane embeddings on the anisotropic background. We have the following system

		t	x	y	z	u	θ	φ	Ω_3
N_c	D3	$\times$	$\times$	$\times$	$\times$				
N_{D7}	D7	$\times$	$\times$	$\times$			$\times$	$\times$	$\times$
N_f	D7	$\times$	$\times$	$\times$	$\times$	$\times$			$\times$

  with $a \propto N_{D7}$.

- We first change the radial coordinate to

$$\rho = \exp \int_{u_H} \frac{-du}{u\sqrt{\mathcal{F} e^{\frac{1}{2}\phi}}}.$$

- This again gives

$$ds^2 = \frac{d\rho^2}{\rho^2} + d\theta^2 + \cos^2\theta d\varphi^2 + \sin^2\theta d\Omega_3^2.$$

- For the induced metric on the D7-brane we have

$$ds^2 = d\rho^2\left(\frac{1}{\rho^2} + \frac{\psi'^2}{1-\psi^2}\right) + (1-\psi^2)d\Omega_3^2,$$

and the D7-brane action reads

$$S_{D7} \propto \int e^{-\phi}\sqrt{-g_{\text{ind}}} \propto \int d\rho \frac{e^{-\frac{1}{2}\phi}}{\rho u^4}\sqrt{\mathcal{F}\mathcal{B}}(1-\psi^2)\sqrt{1-\psi^2+\rho^2\psi'^2}.$$

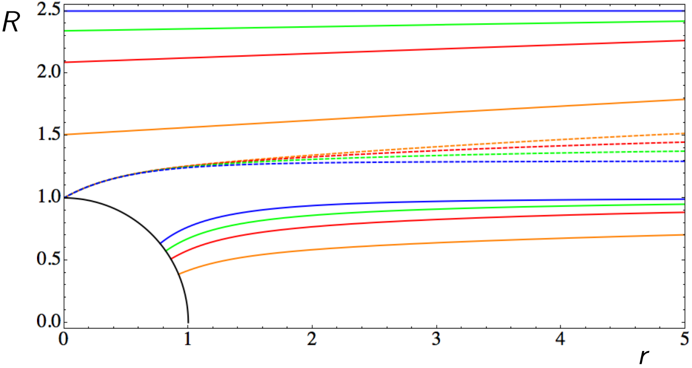


Figure : Profiles for various D7-brane embeddings. Blue, green, red and orange corresponds to $a/T = (0, 6.0, 13.7, 64.1)$ respectively.

- As usual, the on-shell action suffers from divergences which need to be handled using holographic renormalization.
- For the isotropic case, the counterterms are

$$S_{\mathrm{CT}} \propto \frac{1}{4} \int d^4x \sqrt{-\gamma}(1-2\psi^2).$$

- While for the full anisotropic case we need to add

$$
\begin{aligned}
S_{\mathrm{CT}} \propto & \frac{1}{4}\int d^4x\sqrt{-\gamma}e^{\phi}\left[1-\frac{5}{24}e^{2\phi}\partial^i\chi\partial_i\chi-2\psi^2\right] \\
& +\log\epsilon\int d^4x\sqrt{-\gamma}(1-e^{\phi}-\frac{1}{4}e^{2\phi}\partial^i\chi\partial_i\chi[1+\frac{5}{6}e^{\phi}\psi^2] \\
& -\frac{5}{48}e^{5\phi}(\partial^i\chi\partial_i\chi)^2)-\frac{1}{12}\log^2\epsilon\int d^4x\sqrt{-\gamma}e^{5\phi}(\partial^i\chi\partial_i\chi)^2.
\end{aligned}
$$

- We have a counterterm that goes like $\log \epsilon$. That was also needed to renormalize the on-shell action of the anisotropic background itself, which led to the discovery of a conformal anomaly on it.
- We also have a counterterm that goes like $\log^2 \epsilon$, something not previously seen.
- From the renormalized action the quiral condensate is found to be

$$\langle \bar{q} q \rangle \propto c - \frac{5}{24} a^2 m.$$

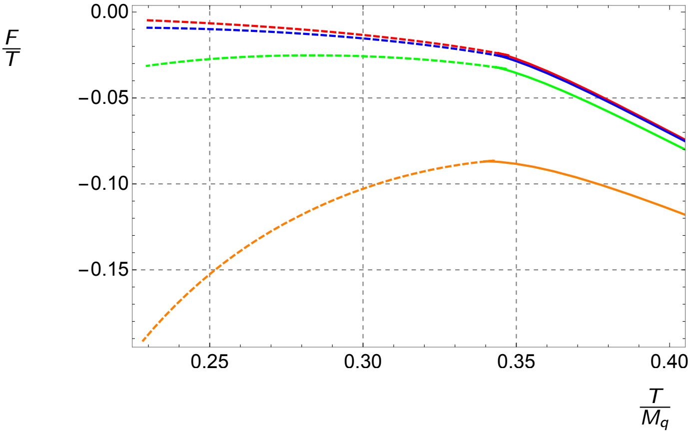


Figure : Free energy as a function of T/M_q for fixed a/T. We hace $a/T = 0.03$ (red), $a/T = 0.22$ (blue), $a/T = 0.55$ (green) and $a/T = 1.37$ (orange).

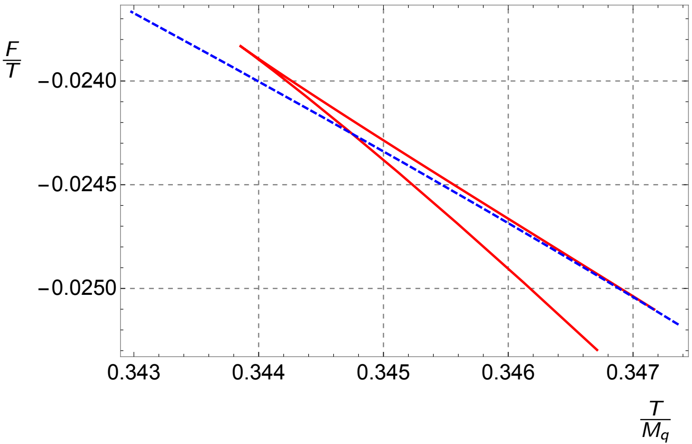


Figure : Free energy as a function of T/M_q for fixed $a/T = 0.03$

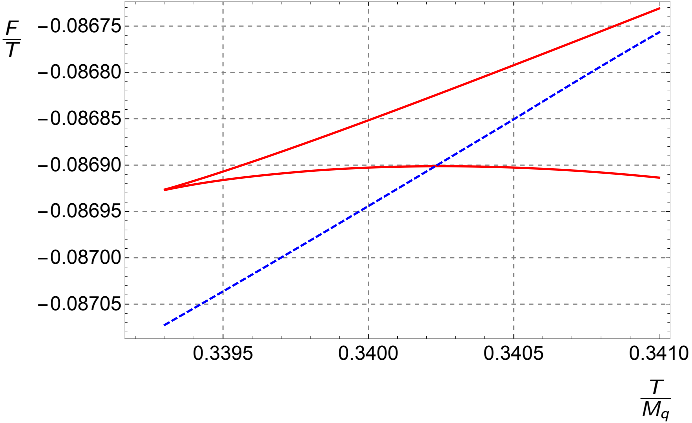


Figure : Free energy as a function of T/M_q for fixed $a/T = 1.37$

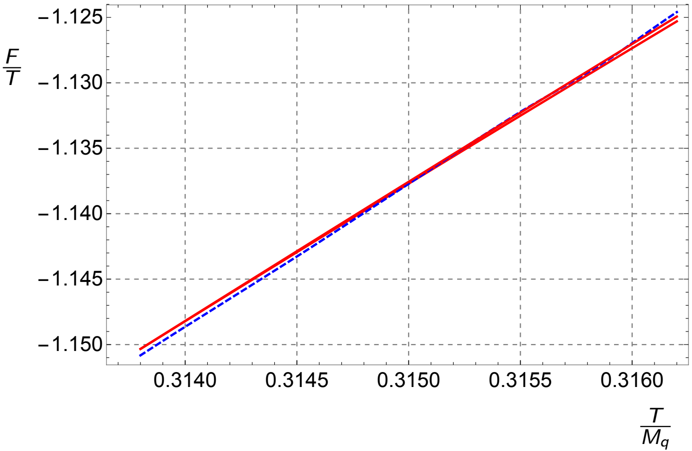


Figure : Free energy as a function of T/M_q for fixed $a/T = 4.41$

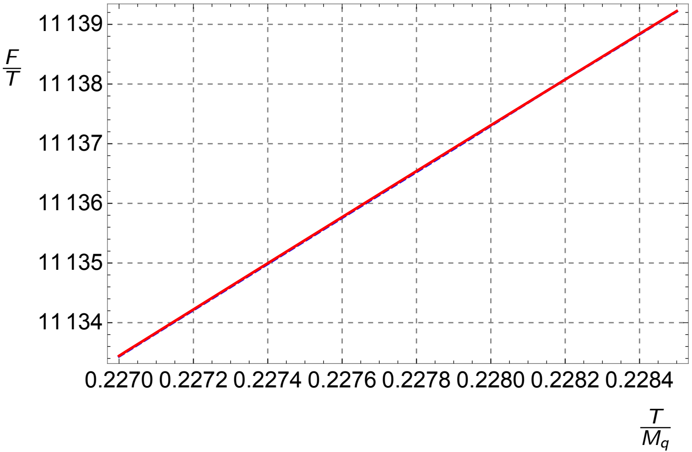


Figure : Free energy as a function of T/M_q for fixed $a/T = 24.9$

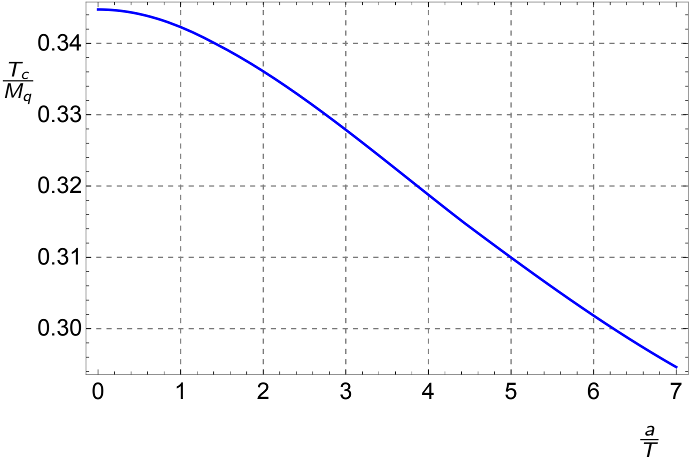


Figure : Critical temperature as a function of the anisotropy.

- For the isotropic case, the authors proved that the system undergoes a first order phase transition analytically, studying excitation supported on the D7-branes.
- Given that the anisotropic geometries are numerical, such an analysis would have been complicated.

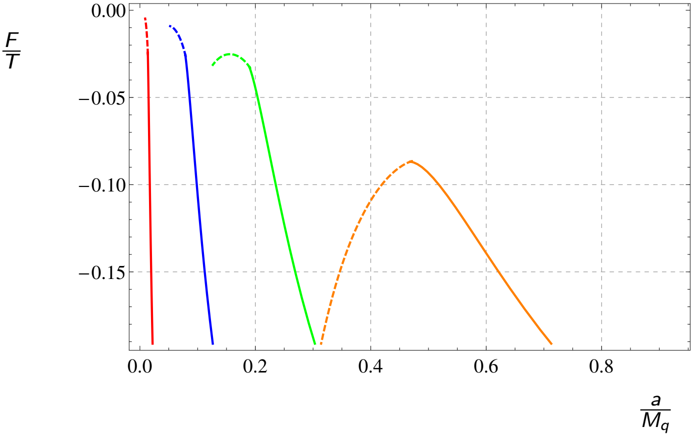


Figure : Free energy as a function of a/M_q for fixed T/a. We have $T/a = 25.3$ (red), $T/a = 4.38$ (blue), $T/a = 1.81$ (green) and $T/a = 0.72$ (orange).

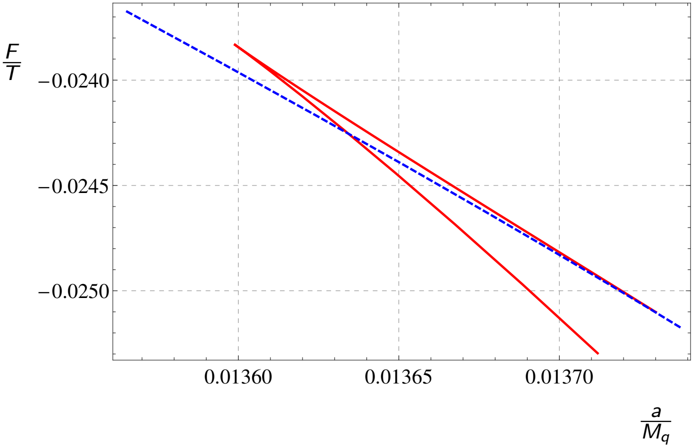


Figure : Free energy as a function of a/M_q for fixed $T/a = 25.3$

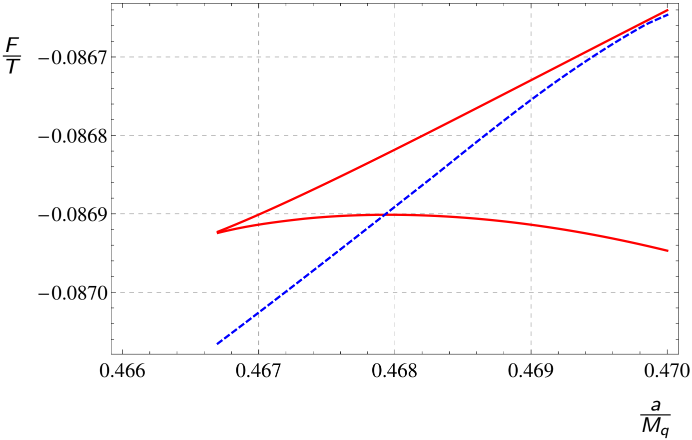


Figure : Free energy as a function of a/M_q for fixed $T/a = 0.72$

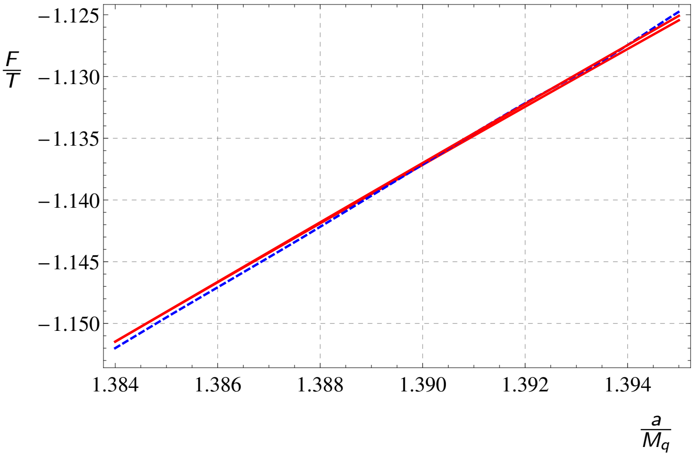


Figure : Free energy as a function of a/M_q for fixed $T/a = 0.22$

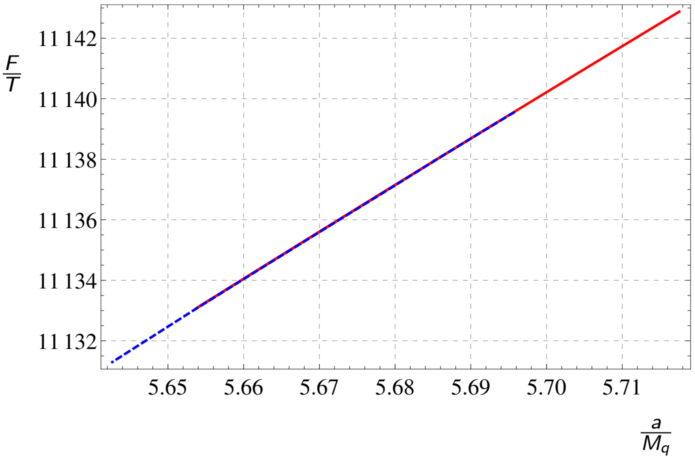


Figure : Free energy as a function of a/M_q for fixed $T/a = 0.04$

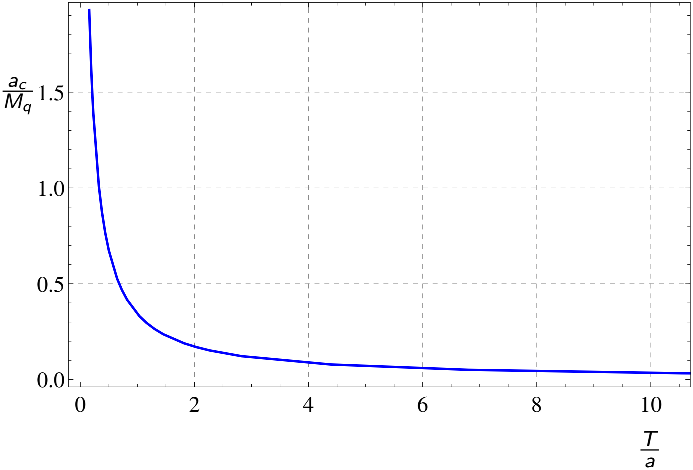


Figure : Critical temperature as a function of the anisotropy.

- This further confirms that the anisotropy has a similar effect as the temperature.
- From the gravity side, it is because the non-trivial axion has the effect to change the gravitational pull.
- From the gauge theory side, it was previously shown that the anisotropy produces an effective force in a preferred direction. One way to understand why it is possible to melt the mesons at a lower temperature its because of these extra forces.

Conclusions

- Flavor degrees of freedom in the boundary gauge theory can be described, at strong coupling, in terms of flavor branes embedded in the dual geometry.
- With this technique, it was previously shown that the fundamental matter added to SYM $\mathcal{N} = 4$ at finite temperature undergoes a first order phase transition at a critical temperature, in which the meson spectrum goes form being discrete to continuous.
- From the gravity point of view, the probe branes goes from a Minkowski embedding, in which they lie entirely outside the horizon, to a black hole embedding, in which they are pulled inside the black hole.

- We performed a similar calculation, embedding probe branes on a geometry dual to SYM $\mathcal{N} = 4$ at finite temperature marginally deformed by a theta term proportional to one of the spatial directions.
- The gravity dual contains a non-trivial axion proportional to one of the spatial directions, with this constant of proportionality being a parameter that measures the degree of anisotropy present.

- Our results shows that the anisotropy has a similar effect as the temperature, causing the phase transition to happen at lower temperatures.
- In addition to a critical temperature, we found that there exist a critical anisotropy at which the phase transition takes place.
- Moreover, we found that increasing the anisotropy smooths out the transition, apparently changing its order.

Thank you for your attention!